Rounding behaviour of professional macro-forecasters

Article

Accepted Version

Clements, M. P. ORCID: https://orcid.org/0000-0001-6329-1341 (2021) Rounding behaviour of professional macro-forecasters. International Journal of Forecasting, 37 (4). pp. 1614-1631. ISSN 0169-2070 doi: https://doi.org/10.1016/j.ijforecast.2021.03.003 Available at https://centaur.reading.ac.uk/96764/

To link to this article DOI: http://dx.doi.org/10.1016/j.ijforecast.2021.03.003

Publisher: Elsevier

Rounding Behaviour of Professional Macro-Forecasters

Michael P. Clements
ICMA Centre,
Henley Business School,
University of Reading

January 21, 2021

Abstract

The rounding of point forecasts of CPI inflation and the unemployment rate by U.S. Professional Forecasters is modest. There is little evidence that forecasts are rounded to a greater extent in response to higher perceived uncertainty about future outcomes. There is clear evidence that probability of decline forecasts are rounded: over a half of the forecast probabilities of decline in the current quarter are multiples of 10. We find that rounding of these probabilities is correlated with worse accuracy, but are cognizant that worse (less accurate) forecasters might round more, rather than the degree of rounding *per se* worsening accuracy. By simulating the loss from rounding for a set of efficient forecasters, we show that the explanation that respondents round otherwise efficient forecasts is untenable, and that the contribution of rounding is of minor importance.

JEL: C53, D84

Rounding, survey expectations, uncertainty, forecast accuracy, histograms.

1 Introduction

In this paper we consider the extent to which *professional* forecasters round their forecasts, why they might round their forecasts, and the impact of this practice on forecast accuracy. A reason for looking at professional forecasts is the striking degree to which *consumers* round their inflation expectations when responding to surveys, as documented by Binder (2017). Unsurprisingly, professional forecasters round their point predictions to a lesser extent. However, surveys of professionals also elicit probability forecasts, and the probability forecasts we consider - probabilities of declines in GDP - do portray clear evidence of rounding.

In response to the question why forecasters round, a leading explanation is that rounding is used to convey uncertainty. In the communication and linguistic theory literature, this is known as *Round Numbers Suggest Round Interpretation* (or RNRI). Binder (2017) documents evidence in support of the RNRI principle in the finance literature, and in surveys of earnings and age, amongst other variables.[1] Provided we can measure perceived uncertainty, we can determine whether uncertainty correlates with rounding. If it does, it seems reasonable to suppose (higher) perceived uncertainty causes rounding, because there is no reason to suppose causality runs in the reverse direction. However, we find little evidence that the degree of rounding is correlated with an agent's perception of the uncertainty she faces. Forecasts do not appear to be rounded to convey uncertainty whether we consider the point predictions or the probabilities of decline in GDP.

Secondly, is rounding benign in terms of forecast accuracy, or are differences between forecasters in terms of forecast accuracy explicable in terms of rounding practices? This is a more difficult question to answer, because greater rounding (for whatever reason) might result in less accurate forecasts, but equally a forecaster may round because she lacks the skill or knowledge to make a more nuanced or precise forecast. When we consider the relationship between rounding and *ex post* forecast accuracy, establishing correlation leaves the question of causation unanswered. As an example, Manski and Molinari (2010) suppose that survey responses might be rounded 'to simplify communication', or 'to convey ambiguity'. According to Manski and Molinari (2010), ambiguity arises when a forecaster feels unable to assign precise probabilities to certain events, such as future inflation or output growth taking on particular values for example, and consequently provides rounded estimates. Hence rounding to simplify communication suggests the respondent could produce a more accurate forecast if she wished, whereas under 'ambiguity' rounding occurs because of a lack of skill. [2]

Is it possible to discriminate between rounding to 'simplify communication' and rounding

[1] Some of the key studies on rounding behaviour are on the reporting of ages of young children in Tanzania by Heitjan and Rubin (1990), as well as cigarette consumption, Wang and Heitjan (2008).

[2] Fischhoff and Bruine De Bruin (1999) refer to 'total ambiguity' as the situation when the forecaster declares 'it's a fifty-fifty chance'.

because the forecaster is unable to make a more precise prediction? For the probabilities of decline, for which there is clear evidence of rounding, we make some modelling assumptions which allow a tentative answer to the question of whether rounding has a significant effect on forecast accuracy. We address this issue by considering whether the forecaster could have made a more accurate non-rounded forecast using information available at the time the forecast was made.[3]

Our main findings are as follows. The professionals' forecasts of CPI and UR (unemployment rate) are rounded to a much lesser extent than consumers' CPI forecasts. There is little evidence that rounding is related to the perceived uncertainty surrounding the outcome, or that rounding negatively impacts forecast accuracy. These findings may in part reflect the fact that any rounding of the CPI and UR forecasts is modest. When we consider the U.S. SPF probability forecasts, namely the probabilities given to the event that quarterly real GDP growth will be negative, there is clear evidence of rounding to multiples of 5 and 10. But as for the point forecasts, there is again little evidence of an association between rounding and perceived uncertainty. We find rounding and forecast performance are correlated - more rounding is associated with less accurate forecasts. We show that forecasters could have made more accurate decline probability forecasts using real-time information, based on their output growth forecasts. However, a simulation study suggests that any rounding of the reported probability of decline forecasts has only a minor effect on the accuracy of the probabilities evaluated as forecasts of the binary event of a decline in output. Moreover, assuming rounding depends on the degree of uncertainty, for example, does not provide a better fit to the actual data, casting doubt on the importance of this putative effect. Finally, we adapt the model of low and high-uncertainty respondents of Binder (2017) to the probability of decline data, and derive some support for the proposition that uncertainty and rounding are positively related.

The plan of the remainder of the paper is as follows. Section 2 briefly reviews approaches to handling rounded data. In section 3 we analyze the rounding behaviour of the professionals' point forecasts of the U.S. CPI inflation and UR forecasts. Section 3.1 begins by describing the survey data, and sections 3.2 and 3.3 provide evidence based on analyzing rounding in aggregate across time, and inter-forecaster variation, respectively. Section 3.4 provides a summary of our findings for professional forecasters compared to the findings in the literature for consumers. In section 4 the probability of decline forecasts are analyzed: section 4.1 considers the inter-forecaster patterns of behaviour, and section 4.2 uses time variation for individual respondents. Section 4.3 relates the probabilities of decline to (simultaneous) output growth forecasts. Section 5 applies the approach of Binder (2017) to the probability of decline data. Section 6 offers some concluding remarks.

[3]This relates to whether the forecasts are efficient, in the sense of Mincer and Zarnowitz (1969): see also Clements (2020b) for an application to the U.S. SPF.

2 Approaches to Handling Rounded Data

Much of the literature dealing with 'coarse' data makes use of multiple imputation (MI) - a simulation-based statistical technique - where the aim is to make 'statistically-valid' inference, in the sense of Rubin (1996). Multiple imputation is commonly used for missing data due to survey non-response, but coarse data refers to any data for which the precise values of the true data are not observed, including rounded or heaped data. Heitjan and Rubin (1990) and Drechsler and Kesl (2016) consider coarse data which is heaped (or rounded), as in the case of self-reported age data. Often the aim is to determine the size and statistical significance of a putative coarsely-observed explanatory variable. Taking the rounded data at face value may not be appropriate.

Following Heitjan and Rubin (1990), Drechsler and Kesl (2016) undertake MI by supposing there is a model for the coarse variable of interest (e.g., that the conditional distribution of the variable is normal given some covariates), and also for the degree of rounding, given that typically rounding may occur to different degrees. The degree of rounding is determined by a latent variable, which is also normally distributed conditional on some covariates. As this variable crosses various thresholds higher degrees of rounding are invoked - the model is an ordered probit over an assumed set of possible degrees of rounding.

Applying MI in the context of analyzing the rounding behaviour of survey respondents would require specifying a model for the variable of interest (the forecast) and for the determinants of rounding behaviour. But surveys of expectations are typically undertaken without recourse to anything other than the forecasts (and actual values) typically because we do not know how the forecasts have been made (in terms of the models, techniques and judgment which have been applied). In most of the paper we do not attempt modelling, but an exception is when the forecast probabilities are related to the output growth forecasts. For the most part we consider what can be learnt from considering the relationships between rounding, and *ex ante* and *ex post* uncertainty, using the reported forecast data (the potentially rounded data), measures derived from the histograms, and the actual data.

Binder (2017) is an attempt to model the forecast generation process and the rounding decision: respondents are assumed to be of two types, either high-uncertainty or low-uncertainty. High-uncertainty forecasters choose rounded responses, while low-uncertainty forecasters choose integer-valued forecasts. That is, she makes the implicit assumption of RNRI - high-uncertainty agents choose rounder responses. In section 5 we apply her approach to the probabilities of decline, but the relatively small number of forecasts at our disposal restricts the application of her approach.

When an individual makes multiple responses to the same survey, or responds to multiple surveys over time, Manski and Molinari (2010) suggest using the pattern of rounding of responses to infer an individual's rounding practice, and provide an algorithm to generate interval data to replace the reported rounded values. This approach was applied to the SPF probability of decline forecasts by Clements (2011).

3 Point Forecasts of Inflation and the Unemployment Rate

3.1 Forecast data and the choice of variable

In this section we describe the forecast data, and provide a first look at the rounding of the point forecasts. We use the U.S. Survey of Professional Forecasters (SPF) as the source of data on professional forecasters. The SPF is a quarterly survey of macroeconomic forecasters of the U.S. economy that began in 1968, administered by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER). Since June 1990 it has been run by the Philadelphia Fed, renamed as the Survey of Professional Forecasters (SPF): see Croushore (1993). The SPF is made freely available by the Philadelphia Fed, allowing results to be readily reproduced and checked by other researchers. Its constant scrutiny is likely to minimize the impact of respondent reporting errors. An academic bibliography of the large number of published papers that use SPF data is maintained[4] and listed 101 papers as of January 2019. As well as providing the point forecasts and probability of decline forecasts which are our main focus, it also provides histogram forecasts which allow the construction of measures of perceived uncertainty.

We consider the CPI inflation forecasts because inflation forecasts have been more extensively studied than perhaps any other variable, and because consumer surveys of inflation are also available, allowing a comparison of the rounding practices of consumers and professional forecasters in section 3.4. We consider the annual fourth-quarter over fourth-quarter CPI inflation forecasts for the current year of the 154 quarterly surveys from 1981:3 to 2019:4. Our sample includes the forecasts from the 127 individuals who responded to a minimum of 12 surveys. The survey also reports these forecasts for the next year, and we analyze these as well. [5] These forecasts are fixed-event in nature, in the sense that the current-year forecasts made in the surveys Q1 to Q4 are of the same target ("event") with a shortening horizon as the year progresses. We also considered the annual unemployment rate forecasts. These are again of the current year and next year, but are the annual averages of the underlying monthly levels. [6]

Throughout the paper, actual values are taken from the Real-Time Data Set for Macroeconomists (RTDSM).[7] For example, for the current year forecasts for the surveys in 2005,

[4] http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/academic-bibliography.cfm.

[5] Also reported are the annualized quarter-over-quarter percent changes of the quarterly average price index level. But annual forecasts, rather than quarterly forecasts, would appear to better match the MSC consumer expectations.

[6] As of 2009:Q2, density projections for the civilian unemployment rate were also elicited.

[7] Available at:

we use the data available in the 2006:Q1 vintage to construct the actual 2005:Q4 on 2004:Q4 inflation rate, and for the annual average 2005 unemployment rate. For the next year forecasts from the same surveys the actuals are taken from the 2007:Q1 vintage of data.[8] However, the revisions to UR and CPI are typically small, compared to the revisions made to the national accounts data (e.g., GDP and the GDP deflator) and so the choice of actual values is likely to be inconsequential. This is not true of the National Income and Product Accounts data such as GDP, which we use subsequently, and which are subject to significant revisions - Clements and Galvão (2019) provide a recent review.

The forecasts were recorded to one decimal place prior to 1990, and thereafter to up to two decimal places. We assume a forecasts is rounded if it is a multiple of 0.5%, and so takes on one of the following distinct values $\{\ldots, 2.0, 2.5, 3.0, 3.5, \ldots\}$. Table 1 records the total number of forecasts of each variable, and the proportion which are expressed as a multiple of 0.5, or 0.1 (i.e., given to one decimal place). If forecasts were recorded to one decimal place, then multiples of 0.5% would be expected to arise 20% of the time if no special significance were attached to such numbers. The observed proportions of multiples of 0.5 (of the forecasts given to one decimal places) are not much greater than this. Even so, we establish in section 3.3 that there are differences between respondents, which we investigate.

Moreover, because forecasts could have been recorded to two decimal places (after 1990), then we would only expect multiples of 0.5 to arise 0.2% of the time, a factor of a hundred less than we observe. Presumably a forecast of 2.51, for example, reflects a model-based forecast which has not been rounded. One might be skeptical of non-model based forecasts which are given to two decimal places. This suggests that rounded forecasts are either judgmental (i.e., non-model based forecasts) or model-based forecasts with judgment applied, and that a preponderance of non-rounded forecasts would suggest unadjusted model forecasts. *De facto* we are seeking to explain the use of judgment (with or without a model) versus automatic forecast generation using a forecasting model or system.

SPF respondents make point forecasts of a range of other variables too. We have selected two headline variables. Both variables are reported to the survey, and recorded in the survey, as percentages (inflation) and rates (unemployment). Other popular choices such as real GDP may be reported as either a level or growth rate, but in either case are recorded as levels. For many purposes this is incidental, but for analyzing rounding behaviour it means that the rounding intended by the respondent may be affected by the calculations undertaken by the Survey.[9]

---

https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data
The RTDSM allows us to use the data vintages which were available at a specific point of time.

[8] For the CPI, the first quarterly data vintage is 1994:Q3, so the actuals up to and including 1994 are from the 1995:Q1 vintage, thereafter they are as described in the text.

[9] See Clements (2015, p. 376) on this point.

The U.S. SPF also provides probability forecasts for a smaller number of variables, in the form of histograms, allowing for the construction of measures of perceived uncertainty. Our assessment of rounding draws on all three types of forecasts. Measures of perceived (or *ex ante*) uncertainty, EAU, have been compared by Clements (2014) (see also Knüppel and Schultefrankenfeld (2019)) to realized or *ex post* uncertainty, EPU. EAU is calculated from the histogram forecasts in advance of the outcome being revealed, whereas *ex post* uncertainty is the squared forecast error (or the MSFE) of the variable in question.[10] EAU will be used to determine whether perceived uncertainty is correlated with rounding behaviour.[11]

Finally, the U.S. SPF provides probabilities of the event that real GDP will decline in the coming quarters, and we use these in section 4.

3.2 Aggregate measures of rounding

We begin by considering an aggregate index of rounding calculated as the proportion of responses to each survey which are a multiple of 0.5, henceforth denoted by the shorthand 'M5'. We run regressions of the time series of the proportion of rounded responses on dummies to denote the quarter of the year of the survey. Given the fixed-event nature of the forecasts, the quarter of the year of the survey determines the forecast horizon. Because uncertainty generally increases in the forecast horizon, the literature discussed in the Introduction suggests the quarter of the survey should be a significant determinant of the degree of rounding if uncertainty and the degree of rounding are related. In addition to the effect from the shortening horizon as the survey quarter moves through the year, we include *ex ante* measures of macro uncertainty, which typically move counter-cyclically. Macro uncertainty at time t is measured by the cross-sectional median of the individual histogram variances of GDP deflator inflation, for the CPI regressions, and by the histogram variances of GDP growth, for the unemployment rate. (We use either the current-year or next-year histograms as appropriate to match the current and next-year CPI and UR forecasts).

The regression results in table 2 for CPI indicate none of the survey-quarter dummies, or macro uncertainty, are significant at the 5% level for the current year. (We include a constant and three dummies, for Q2, Q3 and Q4). This is true for the whole sample, as well as for the various sub-samples we consider.[12] A time trend was included and this is generally negative

[10] We can only measure *ex ante* uncertainty directly for GDP growth and the GDP deflator, as we only have histograms for these two variables. This is a shortcoming which might hinder our ability to detect rounding to convey uncertainty.

[11] If a forecaster's EAU and EPU were closely correlated, it would be difficult to discern whether perceived uncertainty and rounding are correlated, as distinct from forecast performance and rounding being correlated. Clements (2014, Table 5, p.214.) suggests EAU is a poor predictor of EPU across individuals.

[12] The sub-samples are selected as follows. The survey changed hands in 1990:3, and was subsequently administered by the Phildadelphia Fed, with new operating procedures. We consider the periods before and after the change of administration separately. (Engelberg, Manski and Williams (2009) only consider the post 1990 period

and significant when the dependent variable is the proportion of M5 forecasts, suggesting a long term move away from M5. This coincides with more forecasts being reported to two decimal places over the period (for both variables), and when we divide the dependent variable by the proportion of forecasts reported to one decimal place, the size of the coefficient on the trend is reduced, and is insignificant for the period before 1990:4, and after 2005:4.

For the next-year CPI forecasts, the dummy for Q4-surveys is negative and statistically significant (for the whole sample, and for a number of the sub-samples, but not the post-Crisis period), suggesting the degree of rounding is lower (than for the Q1-surveys), consistent with the view that respondents round less when the horizon is shorter. Although as noted, a similar phenomenon is not observed for the current-year CPI forecasts. For the next-year CPI forecasts we also find a significant, positive effect from macro-uncertainty for the whole period when the dependent variable is expressed as a proportion of the forecasts reported to one decimal place.

For the current-year unemployment rate forecasts the survey-quarter effect is insignificant (see table 3), but macro-uncertainty is significantly negative for the period, for both definitions of the dependent variable, which is contrary to the conventional wisdom, and is explored below by exploiting inter-forecaster variation.

For the next year forecasts, there is evidence that the degree of rounding is lower for the Q4 surveys, matching the finding for the next-year CPI forecasts. The Q4 dummy is statistically significant at the 5% level for the whole period, and at the 10% level for some of the sub-periods. But unlike for the next-year CPI forecasts, there is no evidence that macro-uncertainty affects the degree of rounding.

To conclude, the current-year forecasts of both variables do not appear to be influenced by seasonal variation in uncertainty from the fixed-event nature of the forecasts. The rounding of the next-year forecasts is lower for the less uncertain short-horizon Q4 forecasts. Somewhat surprisingly business-cycle variation in macro uncertainty is negatively associated with the rounding of current-year UR forecasts. It is positively associated with the rounding of next-year inflation forecasts.

The evidence based on aggregate data is mixed and does not always point in the same direction. Individuals' rounding behaviour may respond to their own perceptions of the uncertainty of the outlook, and the relationship between uncertainty and rounding may vary across individual. With this in mind, we turn to an individual-level analysis of rounding behaviour.

3.3 Variation across individuals

Figures 1 to 4 depict the cross-sectional variation in the propensity to round, where the propensity to round is calculated as the proportion of the respondent's forecasts which are an exact

as being potentially more reliable). We then split the post-1990 period roughly in half to see whether there were any effects of the Financial Crisis.

multiple of 0.5. Each bar shows the proportion of forecasts rounded by a respondent, and the respondents are ranked by least to most. A horizontal line would indicate no inter-forecaster variation in rounding propensity. For both variables and target periods, there are clear differences across forecasters - some do not round, while those who round the most do so for between 40 and 60% of their forecasts (with the occasional individual who always rounds). We can use the variation in propensity to round across individuals to investigate the relationship between rounding and perceived uncertainty, and forecast accuracy.

As discussed in the introduction, if more able respondents round less, we ought to find that forecasters who round more (as measured by the proportion of their forecasts which are M5) make less accurate forecasts on MSFE. We control for the economic conditions the respondents faced. Otherwise, those who were active during difficult times, such as the 2008-9 period, would have larger uncertainty measures for this very reason. Both our measures of EAU and EPU are calculated as relative measures. For the *ex post* uncertainty, we normalize forecast errors by dividing by the average degree of difficulty experienced in forecasting at that point in time, as measured by the (square root) of the cross-sectional MSFE. This follows D'Agostino, McQuinn and Whelan (2012) and Clements (2014). Specifically, if $e_{i,t}$ denotes the forecast error made by individual i, in response to forecast survey t (the horizon is left unspecified for simplicity), we calculate the normalized forecast errors as:

$$\widetilde{e}_{i,t} = \frac{e_{i,t}}{\sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t} e_{j,t}^2}} \tag{1}$$

where N_t is the number of respondents to survey t. This approach implements an *ex post* adjustment to each forecast error, based on the realized forecast loss, to prevent inter-forecaster comparisons of accuracy being distorted. For each of the 127 respondents, we then calculate MSFE_i as the average squared error of $\widetilde{e}_{i,t}$ over all the surveys t to which individual i responded. For the EAU of respondent i, we first divide the estimate of the histogram variance[13] at time t, denoted $\hat{\sigma}_{i,t}^2$, by the cross-sectional average of all the active participants at t:

$$\widetilde{\sigma}_{i,t}^2 = \frac{\hat{\sigma}_{i,t}^2}{\frac{1}{N_t}\sum_{j=1}^{N_t} \hat{\sigma}_{j,t}^2}$$

and then take the average of $\widetilde{\sigma}_{i,t}^2$ over all the surveys to which i contributed. We have estimates

[13] We estimate the variances by fitting normal distributions to the histograms when non-zero probability mass is assigned to 3 or more (consecutive) intervals, and triangular distributions otherwise (as described by Engelberg *et al.* (2009, p.37-8)). See Clements (2019) for a discussion.

of EAU for output growth and GDP deflator inflation. For both variables the histograms are of the annual growth rates in the current calendar year relative to the previous year, and of the next year relative to the current year. We use the output growth variance estimates to measure EAU regarding the unemployment rate, and the GDP deflator for CPI inflation.[14]

We test for significant correlation between the propensity to round, EAU, and forecast accuracy, allowing that the relationship need not be linear. We consider the relationship between the ranks - whether the individuals who are highly ranked in terms of rounding, are also highly ranked in terms of MSFE, or in terms of their perceived uncertainty. The Spearman rank correlation r lies between -1 and 1, where 0 indicates no relationship. The rank correlation given by:

$$r = 1 - \frac{6R}{N\left(N^2 - 1\right)}$$

where R is the sum of squared differences between the ranks (of the forecasters by degree of rounding, and by the size of MSFE, say).[15]

Table 4 shows some evidence that rounding and forecast performance are negatively associated (i.e., the degree of rounding and MSFE are positively related) for the longer horizon forecasts (the 'next year' forecasts), but not for short horizons. We formally reject the null of no correlation for the UR forecasts (at the 10% level in a two-sided test), but would only reject at the 20% level for the CPI next-year forecasts.[16]

The results based on the cross-sectional variation also suggest that rounding is not related to the perceived uncertainty surrounding the outcome (Table 4, panel B). There is no evidence that perceived and realized uncertainty are correlated across respondents at conventional significance levels for either variable at either horizon: see panel C of table 4. The final panel of table 4 reports the rank correlation of the ratio of the EAU to MSFE with rounding propensity.

[14] Because the CPI inflation forecasts are of Q4 on Q4, the EAU and CPI forecast target periods are not exactly aligned. Unfortunately, histogram forecasts of CPI and UR were only added to the survey in 2007 and 2009, so that using these to derive measures of EAU would dramatically shorten the number of available forecast observations.

[15] It is common to calculate the Fisher transformation,

$$F\left(r\right) = \frac{1}{2}\ln\frac{1+r}{1-r}$$

such that $z = F\left(r\right).\sqrt{\frac{N-3}{1.06}} \sim N\left(0,1\right)$ under the null of statistical independence. As well as reporting r, we report the probability of observing a test statistic less than that obtained under the null hypothesis (of a zero correlation). Probabilities less than 0.05 or greater than 0.95 indicate rejections of the null in a two-sided test at the 10% level. A probability less than 0.05 suggests a negative correlation, and one greater than 0.95 a positive correlation.

[16] We group the current-year forecasts together, and similarly for the next year forecasts. The former have aproximate horizons of 1 to 4-quarters ahead, and the latter of 5 to 8 quarters ahead. In principle one could consider the relationship between rounding and accuracy at a particular horizon, e.g., 8 quarters ahead, if we considered only the next-year forecasts made in response to Q1 surveys. In practice this would mean that the estimates of rounding proportions and MSFE-accuracy would be based on only a quarter as many forecasts.

Clements (2014) finds that forecasters' perceptions of uncertainty (EAU) exceed realized or *ex post* uncertainty at within year horizons, suggesting 'under-confidence'. Beyond one year he finds evidence of over-confidence, as do Binder, McElroy and Sheng (2019) for the ECB Survey of Professional Forecasters. Kenny, Kostka and Masera (2014) report similar findings, as do Glas and Hartmann (2018). Our results suggest no relationship between under/over-confidence and rounding, but we stress that rounding and MSFE are calculated for the individual's point forecasts of CPI and UR, and EAU from the histogram forecasts of the GDP deflator and real GDP growth.[17]

We do not focus on the possible rounding of the histogram forecasts in this paper, but it is worth noting in passing that the effect of rounding on the histogram variances is not clear *a priori*, and will depend on the form the rounding takes. The form of rounding envisaged by Engelberg *et al.* (2009, Appendix, pp.40-1) does not affect EAU. They suppose that rounding can be modelled by considering two cases: in the first they subtract 0.05 of mass from the lowest bin used and put it in a bin immediately above the highest bin used (their ROUND-UP strategy), and in the second, they subtract 0.05 from the highest bin used and add it to the bin below the lowest used (ROUND-DOWN). Their intention is to make an allowance for rounding when they calculate bounds on permissible estimates of the central moments of the histograms, but notice their approach leaves the histogram variance (EAU) unchanged.[18]

In summary, neither the aggregate time-series regressions or the rank correlations between individuals' estimates of rounding propensity and perceptions of forecast accuracy lend unequivocal support to the proposition that professional forecasters round more when they face a more uncertain outlook. There is no evidence that rounding of forecasts of the current year (either CPI or UR) is positively associated with less accurate forecasts.

3.4 Comparison of Professional Forecasters and Surveys of Consumers: Rounding of Point Forecasts

In this section we compare our results on rounding by professional forecasters to those found by Binder (2017) for consumers.

The MSC consumer inflation expectations analyzed by Binder (2017) are surveyed every month, and refer to "the next 12 months".[19] The closest inflation forecasts in the SPF are the

[17] A caveat is that neither the variable definitions (e.g., CPI versus GDP deflator, and UR versus GDP), target periods or horizons line up perfectly, but the evidence is consistent with Clements (2014) where caveats related to different variable definitions do not apply.

[18] As pointed out by a referee, another possibility is that rounded histograms have fewer small probabilities in the more extreme bins, which would reduce EAU. Undoing rounding of this type would further inflate the degree of short-horizon under-confidence found by Clements (2014). Glas and Hartmann (2018) investigate rounding of the histograms and find that respondents who report non-rounded forecasts exhibit less over-confidence than the rounders.

[19] The survey question is phrased as 'By about what percent do you expect prices to go (up/down) on the

annual fourth-quarter over fourth-quarter CPI inflation forecasts for the current year. Although the MSC forecasts are integer-valued, those of the SPF are not, and after 1990 are recorded to two decimal places. Binder (2017) finds that nearly a half of the 219,181 responses to the monthly MSC surveys between January 1978 and December 2013 are multiples of 5%. Perhaps not surprisingly, the respondents to the U.S. Survey of Professional Forecasters report point forecasts of inflation and other variables which are far less coarse, and evidence of rounding is less readily apparent.

Our results also suggest that rounding by professional forecasters is not related to the perceived uncertainty surrounding the outcome: there is no evidence that professional forecasters who round more tend to be individuals with higher measures of *ex ante* forecast uncertainty. This is also at odds with the behaviour of consumers. Using the Federal Reserve Bank of New York Survey of Consumer Expectations (SCE), which provides point predictions and density forecasts of inflation, Binder shows that higher inflation uncertainty, as measured by the inter-quartile range of an individual's inflation density forecast, is positively correlated with the rounding of the point forecast, supporting RNRI for consumers' inflation forecasts.[20]

4 Probability forecasts of Decline in GDP

In this section we consider the relationship between the rounding of the probability of decline forecasts, forecast accuracy, and uncertainty. Clements (2011) has looked at whether rounding accounts for the mismatch documented by Clements (2009) between respondents' probability forecasts of a decline in real output, and the implied probabilities of this event from their histograms for annual real output growth and point forecasts of quarterly output growth. Clements (2011) shows that the mismatch is reduced by allowing for plausible patterns of rounding behaviour (of both the histograms and decline probabilities), but that the overall findings are qualitatively unchanged.

We consider the individual probability of decline forecasts from 1981:3 to 2019:4 surveys. The probability of decline forecast $p_{j,t}^{h}$ is the forecast probability reported by respondent j, to the survey in quarter t, of the event that the level of real output will be lower in quarter $t+h$ than $t+h-1$. The respondents provide forecasts for $h = 0, 1, \ldots 4$, where $h = 0$ refers to a forecast of a decline in output in the current quarter (the survey date quarter) relative to the previous quarter, and $h = 4$ is a forecast of the same quarter a year ahead relative to three quarters ahead.

Table 5 shows that 85% of the current quarter forecasts are reported as multiples of either

average, during the next 12 months?'

[20] Binder (2017) measures 'rounding' as the probability that the forecast has been rounded. The same is true (although the result holds a little less strongly) if a dummy to denote rounding to a multiple of 5 is used instead of the probability of rounding.

5 or 10. This edges up to 88% for the forecasts of a quarterly decline a year ahead. This might suggests little evidence that rounding is done to convey uncertainty: the proportion of longer-horizon forecasts which are rounded is similar to that for the current year forecasts. In addition, the proportion of forecasts which are multiples of 10 is 52 to 55% for all horizons. However, the higher proportion of zero-probability current-quarter forecasts might partly camouflage evidence of rounding to convey uncertainty. Excluding the zero-forecasts leads to an increase in rounding to a multiple of 5 from 64% to 83% as the horizon increases.[21]

Figure 5 ranks each of the 127 forecasters we consider from the individuals who round the least to the two who round all their forecasts to a multiple of 10. The vast majority round between 20% and 80% of their forecasts. We consider whether the inter-forecaster differences in propensities to round are associated with differences in average perceptions of uncertainty. That is, whether some individuals beliefs are persistently more uncertain than those of others. Also of interest is whether the differences in rounding behaviour cause some respondents to be worse than others. The observed differences across individuals in rounding behaviour suggests that an analysis of inter-forecaster variation should be informative about the relationship between rounding and uncertainty.

4.1 Inter-forecaster Variation

To assess forecast accuracy, the probability forecasts are compared to the event of a decline in real GDP calculated from the data vintage available at the time. For example, the current quarter forecasts from the 1981:3 survey are compared to the event of a decline between GDP in 1981:3 and 1981:2, both taken from the 1981:4 data vintage. And the 1981:3 survey $h = 4$ forecasts are compared to the change between 1982:2 and 1982:3 actual values from the 1982:4 vintage of data. We score the probability forecasts using the Brier or quadratic probability score (QPS: Brier (1950)), which is simply the expected squared error $E\left[(p-y)^2\right]$, where p is the probability, and y takes the value of 1 when the event occurs, and zero otherwise.[22] For a sequence of probability forecasts and outcomes, $\{p_t, y_t\}$, $t = 1, \ldots, n$, these scores are calculated as:

$$\text{QPS} = \frac{1}{n}\sum_{t=1}^{n}(p_t - y_t)^2 . \quad (2)$$

[21] One might expect the forecast probability of a decline to approach the relative frequency of declines as the horizon increases, resulting in fewer zero-probability forecasts.

[22] We use QPS rather than the logarithmic probability score (LPS: see Brier (1950) and Good (1952)), defined as $E\left[-y\log(p) - (1-y)\log(1-p)\right]$ because of the occurrence of zero-probability forecasts.

As for the point forecasts, we normalize the QPS for individual i by:

$$\text{QPS}_i = \frac{1}{n_i} \sum_{t \in N_i} \left(\frac{p_{ti} - y_t}{\sqrt{\frac{1}{N_t} \sum_{j=1}^{N_t} (p_{t,j} - y_t)^2}} \right)^2$$

where N_i is the set of surveys (numbering n_i) that i responded to, and N_t is the dimension of the cross-section for survey t.

Table 6 presents rank correlation tests between rounding and accuracy and uncertainty across individuals. We consider rounding to 10, and rounding to 5. Firstly, consider the coarser rounding to 10. If a forecast probability of zero is assumed to reflect a rounded forecast, the table suggests rounding and forecast loss are positively related (i.e., more rounding reduces accuracy) for the longer-horizon $h = 4$ forecasts, but not for the current quarter forecasts. In addition, more perceived uncertainty is associated with less rounding. Treating the zero probability forecasts as being rounded is problematic if they represent underlying beliefs that the event (a decline in output) is extremely unlikely. If we exclude zero probabilities from the definition of rounding, the variation across individuals unambiguously shows rounding is associated with worse forecasts (rounding and QPS are positively related), and removes the anomalous negative relationship between (histogram) uncertainty and rounding. Histogram uncertainty and rounding are not correlated across individuals when zero forecasts are not assumed to have been rounded.

For rounding to 5, rounding is still negatively related to accuracy at both horizons (when rounding excludes zero). We now find that perceived uncertainty is associated with more rounding for the current quarter decline probabilities. Apart from this last finding, the results for rounding to 5 and 10 are the same. In the analysis of individual regressions in the next section we only consider rounding to 10. This is because some respondents always round to 5, making it impossible to analyze rounding behaviour for such individuals at the individual level. Assuming rounding to 10 gives a better balance between rounding and non-rounding than rounding to 5. As shown in table 5, there are few instances of coarser rounding, such as rounding to 25, supporting a focus on rounding to 10.

4.2 Individual Regressions

For the respondents who made a reasonable number of returns we estimate individual regressions. The evidence in section 4.1 based on inter-forecaster variation suggested rounding is not related to uncertainty, but does worsen forecast accuracy. This evidence does not require a linear relationship between uncertainty and rounding. We simply consider whether respondents who round more also have higher (or lower) perceptions of uncertainty than average. Or tend to produce forecasts of higher (or lower) than average accuracy. But we have not exploited the variation in an individual's behaviour over time. For respondents with many forecasts over the period, this variation may be informative. In this section we consider the time variation via individual regressions.

4.2.1 Rounding and Perceived Uncertainty

For each respondent who made at least 40 probability of decline forecasts, we estimated a logit regression for the dummy variable of rounding to 10, with the estimated histogram standard deviation as an explanatory variable. We considered the current-quarter probabilities of decline, and defined uncertainty as the histogram standard deviation of the current-year histogram forecasts, and the four-quarter probability of decline forecasts, using the next-year output growth histograms. We do not report the results, because the uncertainty variable was only statistically significant in a handful of cases, consistent with type 1 error. These findings are consistent with the results based on inter-forecaster variation in table 6, when zero is not treated as a rounded forecast. The individual regression results did not depend on the treatment of the zero probability forecasts.

4.2.2 Rounding and Forecast Accuracy

We regressed the normalized QPS score on a dummy for rounding. Table 7 summarizes the results of the individual regressions for the current-quarter and $h = 4$ quarter ahead probability of decline forecasts, when we do not consider zero forecasts as rounded forecasts. In the Supplementary Materials Appendix we give the individual results which are summarized in table 7, as well as the findings when the rounding dummy takes the value of 1 for a probability of decline forecast of zero: zero forecasts are considered to have been rounded. If we do not consider zero forecasts as rounded forecasts, the rounding dummy is statistically significant for over 60% of the individual respondents (22 of the 36, at the 5% level), for the current-quarter forecasts, and in every such instance is positive, signifying that rounding is associated with less accuracy. If we include zeros as rounded values, the relationship between accuracy and rounding is weakened - the number of regressions in which the dummy variable is statistically significant more than halves. This is consistent with the forecasts of zero reflecting beliefs that the event is extremely unlikely (as opposed to rounding), and this turning out to be accurate. Hence the evidence based on the individual regressions in tables 7 and the table in the Appendix for the current-quarter forecasts is in line with table 6. Forecasts of zero typically reflect correct beliefs that a decline is unlikely, and excluding zero forecasts from the set of rounded forecasts strengthens the finding that rounding has a significant, deleterious effect on forecast accuracy for the majority of the respondents.

For the $h = 4$ quarter ahead forecasts the relationship between rounding and accuracy is weaker, and only statistically significant for 10 of the 36 respondents (at the 5% level).

Rounding is less costly, as might be expected, given that respondents 'true' forecasts of decline four-quarters ahead are likely much less precise than the current-quarter forecasts (i.e., greater ambiguity).

We stress that the dependent variable at time t is the QPS value for individual i at time t divided by the cross-sectional mean of QPS at time t. Hence the coefficient of the rounding dummy records the increase/decrease in i's relative score from rounding.

4.3 Modelling Probability of Decline Forecasts Using Output Growth Forecasts

Some evidence can be brought to bear on whether forecasters round to simplify communication, by considering together the probability of decline and output growth forecasts typically reported by each forecaster. This allows us to approximate an unrounded probability of decline forecast whenever a pair of these forecasts is reported. We suppose that each individual's quarterly growth rate forecast w_{ith} is the mean of a gaussian density forecast, $N\left(w_{ith}, \sigma^2_{w,ith}\right)$, then:

$$p_{ith} = \Pr\left(W_{t+h} < 0 | \mathcal{I}_{it}\right) = \Phi\left(\frac{-w_{ith}}{\sigma_{w,ith}}\right), \tag{3}$$

where $\mathcal{I}_{it}$ denotes individual i's information set at time t, which determines $\left\{w_{ith}, \sigma^2_{w,ith}\right\}$. For the 4th-quarter of the year surveys the histogram forecasts can be used to estimate $\sigma_{w,ith}$, as explained in Clements (2009). For the other quarters of the year this is not possible because of the fixed-event nature of the U.S. SPF histograms. There are ways of calculating approximate fixed-horizon density forecasts (and thus variance estimates), as suggested by Ganics, Rossi and Sekhposyan (2020). A simpler approach may be to estimate the relationship between p and w non-parametrically, using the Nadaraya-Watson regression to estimate the conditional expectation of p given w, $g(w) = E(p|W = w)$. For individual j at survey time t =1991:3, the non-parametric estimator of $g(w)$ is:

$$\hat{p}_{j,t=1991:3,h} = \hat{g}\left(w_{j,1991:3,h}\right) = \frac{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith} - w_{j,1991:3,h}}{b}\right) p_{ith}}{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith} - w_{j,1991:3,h}}{b}\right)} \tag{4}$$

where $k()$ is the kernel function and b the bandwidth.[23] We then extend the model estimation period so that the summation includes 1991:3, and estimate the conditional expectations at

[23] We use Siverman's reference bandwith (or 'rule of thumb'), given by $0.9An^{-0.2}$, where A is the smaller of the sample standard deviation, and the interquartile range divided by 1.34, and n is the number of observations: (Silverman (1986)). The reported results are based on a triangular kernel, although the results are not sensitive to use of a Gaussian kernel. See the note to table 7

$w_{j,1991:4,h}$, for each j, [24] and so on. The limits of the summations in (4) indicate that the history of all respondents' (and not just j's) forecasts enter the calculation. This is because of the relatively small number of forecasts for any one respondent. We also assume that past data is as relevant as more recent data: this could be relaxed by estimating the model on a rolling window of data (as opposed to the recursive forecasting scheme we have adopted).

When the reported forecasts p are rounded, they measure the 'true' forecasts, say, p^0, with error, $p_{ith} = p^0_{ith} + r_{ith}$, say. Equation (4) can be viewed as a model of the relationship between the true unrounded forecasts p_0 and w if we substitute $p_{ith} = p^0_{ith} + r_{ith}$ into (4) and assume that the term:

$$\frac{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith} - w_{j,1991:3,h}}{b}\right) r_{ith}}{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith} - w_{j,1991:3,h}}{b}\right)}$$

is negligible and can be ignored. Whereas ignoring rounding (taking the reported value at face value) may give misleading results when the object of the analysis is to learn about the size and significance of an explanatory variable in a structural model, our aim is more modest, to obtain a simple forecasting model for p^0.

The last two columns of table 7 summarize the findings across individuals regarding the ratio of the QPS for $\hat{p}$ to that for p. The results pertain to the subset of forecasts t between 1991:3 and 2019:4 for which both $\hat{p}_{it}$ and p_{it} exist for respondent i.[25] $\hat{p}$ is more accurate on QPS than p more often than not: the current-quarter predicted probabilities are more accurate for 27 of the 36 respondents, and over 10% more accurate for nearly half of the respondents (16 of the 36).[26]

These findings are consistent with the proposition that probability of decline forecasts are rounded to simplify communication. This is because more accurate non-rounded forecasts were readily available, based only on the individual's quarterly output growth forecasts, and a simple model linking the two (which can be estimated from the history of w and p forecasts through $t-1$). That is, using only information available to the forecaster at the time the forecast was made, superior (non-rounded) forecasts could have been made.

However it may be wrong to attribute the difference in accuracy between $\hat{p}$ and p solely to the effect of rounding. It may be that respondents who are less good forecasters tend to round more. Suppose that $\hat{p} = g(W = w)$ exploits useful information neglected by the respondent in

[24] Hence the approach is real time in the sense that at each t, the forecast $\hat{p}_{jth}$ uses forecasts of w and p through $t-1$, which will be known to each respondent, and the respondent j's time t forecast w_{jth}. If $\hat{p}_{jth}$ is superior to p_{jth} in terms of forecast accuracy, then the reported forecasts are inefficient in the sense that they do not exploit all readily available information: see Mincer and Zarnowitz (1969).

[25] The first survey is 1991:3 because the 1981:3 to 1991:2 period is used to estimate the relationship beween the forecast of quarterly output growth w_{it} and the forecast probability of decline p_{it}. Note that p_{ith} will typically be missing for some periods, and $\hat{p}_{ith}$ will be missing whenever w_{ith} is missing.

[26] We consider QPS as the simplest way of scoring the forecast probabilities: Lahiri and Wang (2013) consider other approaches.

producing p, such that $\hat{p}$ would be more accurate than the reported p even if the latter were not rounded. To address this issue, we use a simulation to estimate the effects of rounding on the accuracy of the forecast probabilities of decline. In the simulation, we can compare the true probabilities (denoted by p^0) to the rounded probabilities, p, assuming a particular rounding scheme, in terms of QPS: i.e., we can compare $E\left(p^0 - 1_{w<0}\right)^2$ and $E\left(p - 1_{w<0}\right)^2$. The difference between the simulation estimates of these two losses estimates the effect of rounding. Because we only observe the reported forecast probabilities, the empirical counterpart of this loss is not available. In the simulation we also estimate the difference between $E\left(p - 1_{w<0}\right)^2$ and $E\left(\hat{p} - 1_{w<0}\right)^2$, where $\hat{p}$ is the estimator of $E(p|W = w)$. Empirical estimates of the ratio of these two losses are reported in table 7.

Details of the data generating process are provided in the section 7 Appendix. In brief, forecaster behaviour is given by the noisy information model,[27] and is loosely calibrated on SPF forecasts of quarterly real GDP. Probability of decline forecasts are obtained assuming gaussianity and assuming each respondent forecasts the variance of future output growth. We allow the forecast error variance to differ across respondents by assuming different signal precisions.[28] The model assumes forecasters are rational given the informational rigidities they face. Our simulation findings were qualitatively unaffected if instead of assuming noisy information, we allow that past values of real output growth are observed (termed private information), or if we suppose that respondents are no longer rational but are subject to a behavioural bias, such as in the diagnostic expectations of Bordalo *et al.* (2020).

We investigate the consequences of three rounding behaviours. In the first, R_1, agents round to a given multiple with a prescribed probability. We assume rounding to a multiple of 0.1 with a probability of 0.4. The second R_2 supposes the probability of rounding depends on the agent's true probability, p^0, the simplest case of which is to assume the probability of rounding equals p^0. Recessionary times are typically associated with higher uncertainty, so that when the probability of a decline is higher the agent is more likely to round her forecast to reflect the higher uncertainty. Thirdly, R_3, when we assume heterogeneous agents, the probability of rounding is higher for agents with higher forecast-error variances (that is, with less precise signals). Specifically, we assume the probability of rounding is 0.4 times the ratio of the agent's forecast-variance to the median error-variance across agents. Hence the median forecaster acts as under our first assumption, whereas better (worse) forecasters are less (more) likely to round.[29] R_3 captures the idea that agents round to convey uncertainty: in our model

[27] This noisy information model has become one of the leading models of forecast behaviour, see e.g., Coib08

[28] See, e.g., D'Agostino *et al.* (2012) and Clements (2020b, 2020a) for evidence on whether some forecasters really are better than others. In terms of the noisy information model, better forecasters are the recipients of more precise signals. Bordalo, Gennaioli, Ma and Shleifer (2020) suggest that instead of interpreting the noisy signal an agent receives as reflecting 'inattentiveness', as is often done in this literature, it might be more reasonable to suppose it reflects the use of "different models or pieces of evidence".

[29] In our simulations the forecast-error variances differ across agents when we allow heterogeneity, but do not

they correctly perceive the uncertainty they face.

The findings of the simulations are recorded in table 8. The column headed $E\left(p^0 - 1_{w<0}\right)^2$ reports the estimate of the QPS value for the true forecast probabilities, and the next three columns are the ratios of the QPS values for each of the three rounded forecasts (R_1, R_2 and R_3), to that using p^0. [30] We report the cross-sectional mean across the respondents as well as the minimum and maximum. When agents are homogeneous the three measures are close to each other, and only differ because of simulation error. The effects of rounding on event-forecast accuracy are barely perceptible, and any dependence on the precision of the signal is again barely perceptible. Because the empirical estimates are often of an order of magnitude larger than the estimates in the simulation, we conclude that the difference in accuracy between $\hat{p}$ and p for the most part does not reflect rounding, but forecast inefficiency.

Whereas rounding has only a small effect on forecasting the event that output will be lower, the simulation shows that it naturally has a much larger effect when we compare the accuracy of $\hat{p}$ and p as forecasts of p^0. The last three columns show the expected squared error of the corrected probability $\hat{p}$ (for a given rounding scheme, R_j) as a forecast of p^0 as a ratio of the expected squared error of p (for a given rounding scheme, R_j) as a forecast of p^0. The corrected forecasts are obtained from (4) by replacing p_{ith} with the forecasts obtained by applying one of the three rounding schemes. The last three columns show that on average across respondents the corrected forecasts can be much more accurate than that of the rounded forecasts for the true probabilities when the respondents are homogeneous (rows with 'Hetero. = 0'). Under heterogeneity, and when the private signals are relatively more important ($\sigma_\varepsilon = 1$, as opposed to 3), the performance of the non-parametric estimate of the respondent's unrounded probability p^0 depends on the precision of the private-information signal. For example, under noisy information the range for 'random' rounding R_1 is 0.131 to 4.058. The maximum value of 4.058 is found for the forecaster who receives the most precise signal. This respondent's rounded forecasts are more accurate than the non-parametric estimates, because of the importance of the time-t signal, which is foregone when (4) is used. As the signal precision deteriorates, the omission of the signal in estimating the non-rounded value matters less, and the ratio falls. The same is observed for the other rounding schemes.

The results for the three different expectations structures (noisy and private information, and diagnostic expectations) are qualitatively in line with one another.

To recap: we cannot directly determine the effect of rounding on event-forecasting accuracy because we only observe the reported value, which has been rounded to an unknown extent. However, our simulation shows that empirically-plausible rounding behaviours only worsen

depend on time.

[30] We report the averages of the 100 respondents, where for each respondent we calculate QPS over 10,000 replications. When the forecasters are homogeneous, this is equivalent to 1,000,000 replications of one respondent, but the distinction is meaningful when we permit heterogeneity.

accuracy by less than half a percentage point. Hence the improvements in empirical forecast accuracy of the order of 10% from the non-parametric estimates $\hat{p}$ likely reflect the failure of the forecasters to fully utilize all available information, rather than the effects of rounding *per se*. That is, differences in event forecast accuracy across forecasters reflects differences in forecasting ability.

5 Model of the Cross-sectional Distribution of Probability of Decline Responses

Binder (2017) supposes reported survey responses R_{it} are generated by a combination of lower-uncertainty l-agents, who round less, and higher-uncertainty h-agents, who round more. (Note h now indicates 'high', not the forecast horizon). She uses the proportion of the two types at each time t as the basis of a time-series uncertainty index. We adapt her analysis of CPI inflation rates for the probabilities of decline.

We suppose type-h respondents choose from a set of rounded responses, S_h, and type l from a set S_l, where $S_h \subset S_l$, indicating S_h is coarser than S_l. The distribution of survey responses at t is a mix of the probability mass function (pmf) ϕ_t^l for type l-agents, with support on S_l, and ϕ_t^h, with support on S_h. For the probability forecasts, $S_l = \{\ldots, 0, 1, 2, 3, \ldots\}$ and $S_h = \{0, 5, 10, 15, \ldots\}$, where most but not all reported forecasts are integer-valued. If R_{it} is not an element of S_h, then i is not type h, but if $R_{it} \in S_h$, i could be either type. We assume the pdf of i is either $p_l(x)$ if $i \in l$ or $p_h(x)$ if $i \in h$, where $p_l(x) \sim LG(\mu_l, \sigma_l)$, and $p_h(x) \sim LG(\mu_h, \sigma_h)$, where we expect to find $\sigma_h > \sigma_l$ if uncertainty explains rounding. That is, the agents who round are those who perceive more uncertainty. LG denotes the logistic pdf density function, and replaces the normal density used by Binder. Hence:

$$\begin{aligned}
\phi_t^l &= P(R_{it} = j | i \in l) = \int_{f_{\min}^l(j)}^{f_{\max}^l(j)} p_l(x)\, dx,\ j \in S_l \\
\phi_t^h &= P(R_{it} = j | i \in h) = \int_{f_{\min}^h(j)}^{f_{\max}^h(j)} p_h(x)\, dx,\ j \in S_h
\end{aligned}$$

where $f_{\min}^l(j)$ and $f_{\max}^l(j)$ are the min and max values of the underlying forecast distribution that are rounded to the reported value j if the forecaster is type l, and similarly for $f_{\min}^h(j)$ and $f_{\max}^h(j)$ for type h. If we consider rounding to '5', for example, then for $j = 45$, say, $\left[f_{\min}^h(j = 45), f_{\max}^h(j = 45)\right] = [42.5, 47.5]$, and $\left[f_{\min}^l(j = 45), f_{\max}^l(j = 45)\right] = [44.5, 45.5]$. For j which is not a multiple of 5, say, $j = 46$, $\left[f_{\min}^l(j = 46), f_{\max}^l(j = 46)\right] = [45.5, 46.5]$, but $\phi_t^h(j = 46) = 0$.

In period t the survey responses come from $\phi_t = \lambda_t \phi_t^h + (1 - \lambda_t)\, \phi_t^l$, and maximizing the log-likelihood $\sum_{j \in S_l} N_{tj} \log \phi_t(j)$ provides estimates of $\{\lambda_t, \mu_{t,h}, \sigma_{t,h}, \mu_{t,l}, \sigma_{t,l}\}$, where N_{tj} is the number of responses $R_{it} = j$, $j \in S_l$, at time t. The number of SPF responses at each t are far fewer than those available to Binder, and so we aggregate the responses over t, and suppose $\phi_t^h = \phi^h$, $\phi_t^l = \phi^l$ and $\lambda_t = \lambda$, for all t, as well as $N_j = \sum_t N_{tj}$. This means it is not possible to consider time-series variation - for example, how the proportion of rounders evolves over time. But within this approach we can ask whether rounders perceive more uncertainty than the non-rounders, and formally test (via a standard likelihood ratio test) the null hypothesis that $\sigma_h = \sigma_l$. Figure 6 plots the estimated densities values $\hat{\phi}_t^l$ and $\hat{\phi}_t^h$, and records the estimates of the location and scale parameters of the logistic distributions. We find that $\hat{\sigma}_l < \hat{\sigma}_h$, and reject the null of equality of the σ's at any significance level. We find $\hat{\lambda} = 0.59$, indicating that over a half of the respondents are of the high-uncertainty types who tend to round their forecasts.

While our findings are consistent with those of Binder for consumers, because there are too few observations to estimate λ and the parameters of the two distributions for each t, we are unable to correlate a series of λ estimates with proxies of forecast uncertainty, and are not able to show whether or not rounding depends on uncertainty.

6 Conclusions

There is some evidence that U.S. Professional Forecasters round their point forecasts of CPI inflation and the unemployment rate, but perhaps not surprisingly to a lesser extent than Binder (2017) found for consumers' inflation forecasts. We found little evidence that forecasts were rounded to a greater extent in response to higher perceived uncertainty about future outcomes, at odds with the findings for consumer inflation forecasts. Hence our findings suggest that the degree of rounding of point predictions by professional forecasters could not serve as a proxy for perceived uncertainty, contrary to the findings of Binder (2017). However, there is some evidence that respondents who are more prone to round their forecasts produce less accurate forecasts.

By way of contrast, the event-probability forecasts (the forecast probabilities of a decline in output) are clearly rounded. Around 85% of the probabilities that output will decline in the current quarter are reported as multiples of 5, and over a half are multiples of 10. If we consider inter-forecaster variation, or individual regressions of accuracy on rounding, we find that rounding is correlated with worse event-forecast accuracy. However, our findings might reflect the fact that worse (less accurate) forecasters round more, rather than the degree of rounding *per se* worsening accuracy.

A respondent's probability of decline and output growth forecasts of the same quarter ought to be closely related, and we exploit this to generate series of non-rounded probability forecasts for all the respondents. These are found to be up to 10% more accurate on QPS than the reported probabilities for over a half of respondents. If respondents' behaviour can be approximated by our model (and they make use of the information on output growth forecasts), then subsequent rounding of these 'efficient' forecasts accounts for the reduction in accuracy. But respondents' forecasts may not be efficient, and less good forecasters may round more.

We overcome this impasse by simulating the loss from rounding for a set of efficient forecasters, under a number of assumed rounding schemes. The size of the simulated losses from rounding are not commensurate with the empirical estimates. Rounding of itself has a relatively minor impact on *event*-forecast accuracy, and the assumption that respondents round otherwise efficient forecasts is untenable.

We conclude that rounding of probability forecasts appears to matter little for event-forecasting. It would matter if the rounded forecasts were compared to the true probabilities, but the latter are of course only available in simulation studies.

We have not considered rounding probabilities in the form of survey histogram forecasts, and leave this issue for future research.

References

Binder, C., McElroy, T., and Sheng, X. (2019). The Term Structure of Uncertainty: New Evidence from Survey Expectations. Discussion paper.

Binder, C. C. (2017). Measuring uncertainty based on rounding: New method and application to inflation expectations. *Journal of Monetary Economics*, *90*(C), 1–12.

Bordalo, P., Gennaioli, N., Ma, Y., and Shleifer, A. (2020). Overreaction in Macroeconomic Expectations. *American Economic Review*, *110*(9), 2748–2782.

Brier, G. W. (1950). Verification of forecasts expressed in terms of probability. *Monthly Weather Review*, **75**, 1–3.

Clements, M. P. (2009). Internal consistency of survey respondents' forecasts: Evidence based on the Survey of Professional Forecasters. In Castle, J. L., and Shephard, N. (eds.), *The Methodology and Practice of Econometrics. A Festschrift in Honour of David F. Hendry. Chapter 8*, pp. 206–226. Oxford: Oxford University Press.

Clements, M. P. (2011). An empirical investigation of the effects of rounding on the SPF probabilities of decline and output growth histograms. *Journal of Money, Credit and Banking*, **43(1)**, 207–220.

Clements, M. P. (2014). Forecast Uncertainty - Ex Ante and Ex Post: US Inflation and Output Growth. *Journal of Business & Economic Statistics*, **32(2)**, 206–216. DOI: 10.1080/07350015.2013.859618.

Clements, M. P. (2015). Are professional macroeconomic forecasters able to do better than forecasting trends?. *Journal of Money, Credit and Banking*, **47,2-3**, 349–381.

Clements, M. P. (2019). *Macroeconomic Survey Expectations*: Palgrave Texts in Econometrics. Palgrave Macmillan. DOI 10.1007/978-3-319-97223-7.

Clements, M. P. (2020a). Are some Forecasters' Probability Assessments of Macro Variables Better Than those of Others?. *Econometrics*, **8, 16**.

Clements, M. P. (2020b). Forecaster efficiency, accuracy and disagreement: evidence using individual-level survey data. *Journal of Money, Credit and Banking*. Forthcoming.

Clements, M. P., and Galvão, A. B. (2019). Data revisions and real-time forecasting. *The Oxford Research Encyclopedia of Economics and Finance*. Oxford University Press. doi:10.1093/acrefore/9780190625979.013.248.

Croushore, D. (1993). Introducing: The Survey of Professional Forecasters. *Federal Reserve Bank of Philadelphia Business Review, November*, 3–15.

D'Agostino, A., McQuinn, K., and Whelan, K. (2012). Are some forecasters really better than others?. *Journal of Money, Credit and Banking*, **44(4)**, 715–732.

Drechsler, J., and Kesl, H. (2016). Beat the Heap: An Imputation Strategy for Valid Inferences from Rounded Income Data. *Journal of Survey Statistics and Methodology*, *4*(1), 22–42.

Engelberg, J., Manski, C. F., and Williams, J. (2009). Comparing the point predictions and subjective probability distributions of professional forecasters. *Journal of Business and Economic Statistics*, **27(1)**, 30–41.

Fischhoff, B., and Bruine De Bruin, W. (1999). Fifty Fifty = 50%?. *Journal of Behavioral Decision Making*, *12*(2), 149–163.

Ganics, G., Rossi, B., and Sekhposyan, T. (2020). From fixed-event to fixed-horizon density forecasts: Obtaining measures of multi-horizon uncertainty from survey density forecasts. Discussion paper.

Glas, A., and Hartmann, M. (2018). Overconfidence versus Rounding in Survey-Based Density Forecasts. Discussion paper. Available at SSRN: https://ssrn.com/abstract=3202810 or http://dx.doi.org/10.2139/ssrn.3202810.

Good, I. (1952). Rational decisions. *Journal of the Royal Statistical Society. Series B*, **14 (No. 1)**, 107–114.

Heitjan, D. F., and Rubin, D. B. (1990). Inference from Coarse Data via Multiple Imputation with Application to Age Heaping. *Journal of American Statistical Association*, *85*(410), 304–314.

Kenny, G., Kostka, T., and Masera, F. (2014). How Informative are the Subjective Density Forecasts of Macroeconomists?. *Journal of Forecasting*, *33*(3), 163–185.

Knüppel, M., and Schultefrankenfeld, G. (2019). Assessing the uncertainty in central banks' inflation outlooks. *International Journal of Forecasting*, **35**, 1748–1769.

Lahiri, K., and Wang, J. G. (2013). Evaluating probability forecasts for GDP declines using alternative methodologies. *International Journal of Forecasting*, *29*(1), 175–190.

Manski, C. F., and Molinari, F. (2010). Rounding probabilistic expectations in surveys. *Journal of Business and Economic Statistics*, **28:2**, 219–231.

Mincer, J., and Zarnowitz, V. (1969). The evaluation of economic forecasts. In Mincer, Jacob, A. (ed.), *Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance*, pp. 3–46. New York: National Bureau of Economic Research.

Rubin, D. B. (1996). Multiple imputation after 18+ years. *Journal of the American Statistical Association*, *91*(434), 473–489.

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.

Wang, H., and Heitjan, D. F. (2008). Modeling Heaping in Self-Reported Cigarette Counts. *Statistics in Medicine*, *27*(19), 3789–3804.

Table 1: Rounding of CPI and UR annual point forecasts

	no. forecasts. (#)	M5/#	M1/#
CPI current year (Q4 on Q4)	4709	0.170	0.733
CPI next year (Q4 on Q4)	4432	0.193	0.783
UR current year	4885	0.175	0.765
UR next year	4757	0.176	0.785

M5 denotes the forecast is a multiple of 0.5, and M1 a multiple of 0.1 (that is, reported to one decimal place). The last two columns are the proportions of forecasts which are M5 and M1 respectively.

Table 2: Aggregate time-series rounding results for CPI

	Constant	Q2	Q3	Q4	Time Tr.	Macro Unc.	R^2
Current year (Q4 on Q4)							
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.348	0.000	0.013	0.005	-0.002	-0.064	0.409
	0.000	0.999	0.512	0.833	0.000	0.225	.
1981:3 to 1990:3	0.315	0.067	0.060	-0.033	-0.002	-0.049	0.117
	0.012	0.239	0.381	0.691	0.204	0.680	.
1990:4 to 2019:4	0.398	-0.021	-0.004	0.011	-0.002	-0.112	0.460
	0.000	0.213	0.844	0.648	0.000	0.096	.
1990:4 to 2005:4	0.451	-0.027	-0.018	0.019	-0.003	-0.141	0.244
	0.000	0.366	0.565	0.669	0.001	0.085	.
2006:1 to 2019:4	0.303	-0.014	0.017	0.012	-0.002	0.014	0.333
	0.003	0.447	0.295	0.535	0.005	0.915	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.269	0.006	0.029	0.038	-0.001	0.002	0.108
	0.000	0.782	0.243	0.175	0.008	0.971	.
1981:3 to 1990:3	0.315	0.073	0.066	-0.037	-0.002	-0.060	0.126
	0.012	0.199	0.344	0.657	0.306	0.603	.
1990:4 to 2019:4	0.289	-0.016	0.008	0.047	-0.001	-0.034	0.121
	0.000	0.462	0.753	0.125	0.021	0.652	.
1990:4 to 2005:4	0.426	-0.027	-0.021	0.023	-0.002	-0.123	0.185
	0.000	0.405	0.528	0.622	0.008	0.137	.
2006:1 to 2019:4	0.191	-0.007	0.042	0.083	0.000	0.170	0.138
	0.290	0.834	0.254	0.052	0.728	0.423	.
Next year (Q4 on Q4)							
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.300	-0.027	-0.019	-0.055	-0.001	0.044	0.336
.	0.000	0.221	0.410	0.013	0.000	0.246	.
1981:3 to 1990:3	0.129	-0.024	-0.008	-0.028	0.001	0.150	0.134
.	0.265	0.689	0.899	0.646	0.553	0.049	.
1990:4 to 2019:4	0.365	-0.033	-0.027	-0.063	-0.001	-0.030	0.294
.	0.000	0.157	0.293	0.008	0.000	0.633	.
1990:4 to 2005:4	0.367	-0.061	-0.054	-0.080	-0.001	-0.041	0.124
.	0.000	0.100	0.210	0.037	0.126	0.605	.
2006:1 to 2019:4	0.215	-0.002	0.003	-0.042	-0.001	0.067	0.167
.	0.082	0.933	0.904	0.146	0.352	0.511	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.203	-0.019	-0.015	-0.036	0.000	0.103	0.091
.	0.000	0.463	0.568	0.194	0.672	0.014	.
1981:3 to 1990:3	0.087	-0.008	-0.005	-0.024	0.002	0.170	0.142
.	0.526	0.901	0.931	0.702	0.398	0.049	.
1990:4 to 2019:4	0.258	-0.028	-0.023	-0.042	0.000	0.016	0.024
.	0.000	0.335	0.436	0.177	0.939	0.814	.
1990:4 to 2005:4	0.337	-0.061	-0.050	-0.077	-0.001	-0.031	0.089
.	0.001	0.119	0.263	0.057	0.468	0.699	.
2006:1 to 2019:4	-0.162	0.008	0.013	0.009	0.002	0.332	0.048
	0.510	0.849	0.732	0.860	0.142	0.084	.

For each sample period, the first row gives the parameter estimates, and the second row the p-value of the null that the coefficient equals zero, using heteroscedasticity-consistent standard errors.

Table 3: Aggregate time-series rounding results for UR

	Constant	Q2	Q3	Q4	Time Tr.	Macro Unc.	R^2
Current year							
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.420	-0.021	-0.038	-0.043	-0.002	-0.203	0.112
.	0.000	0.384	0.287	0.456	0.000	0.005	.
1981:3 to 1990:3	0.063	0.014	0.069	0.209	0.001	0.098	0.093
.	0.768	0.815	0.512	0.293	0.739	0.598	.
1990:4 to 2019:4	0.476	-0.014	-0.026	-0.063	-0.002	-0.220	0.155
.	0.000	0.652	0.583	0.359	0.000	0.047	.
1990:4 to 2005:4	0.376	0.003	-0.017	-0.030	0.000	-0.286	0.046
.	0.017	0.943	0.805	0.776	0.853	0.039	.
2006:1 to 2019:4	0.108	-0.001	0.022	-0.012	0.000	0.098	0.052
.	0.276	0.978	0.713	0.884	0.555	0.526	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.377	-0.017	-0.032	-0.039	-0.001	-0.172	0.030
.	0.000	0.582	0.456	0.567	0.042	0.031	.
1981:3 to 1990:3	0.062	0.014	0.074	0.209	0.001	0.098	0.092
.	0.771	0.816	0.488	0.293	0.727	0.600	.
1990:4 to 2019:4	0.424	-0.011	-0.027	-0.067	-0.001	-0.196	0.042
.	0.000	0.786	0.639	0.411	0.042	0.115	.
1990:4 to 2005:4	0.297	0.013	0.000	-0.003	0.001	-0.261	0.052
.	0.085	0.791	0.995	0.978	0.685	0.074	.
2006:1 to 2019:4	0.143	0.003	0.015	-0.028	0.000	0.171	0.044
	0.452	0.973	0.896	0.858	0.879	0.574	.
Next year							
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.290	-0.024	-0.026	-0.042	-0.001	0.000	0.305
.	0.000	0.146	0.163	0.011	0.000	0.983	.
1981:3 to 1990:3	0.237	-0.055	-0.040	-0.073	-0.001	0.050	0.179
.	0.015	0.131	0.380	0.077	0.768	0.256	.
1990:4 to 2019:4	0.319	-0.019	-0.027	-0.034	-0.001	-0.037	0.255
.	0.000	0.300	0.206	0.059	0.000	0.132	.
1990:4 to 2005:4	0.235	-0.033	-0.051	-0.045	0.000	-0.022	0.076
.	0.000	0.204	0.083	0.077	0.557	0.437	.
2006:1 to 2019:4	0.147	-0.004	-0.001	-0.021	0.000	-0.009	0.025
.	0.069	0.858	0.982	0.349	0.826	0.893	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.221	-0.024	-0.022	-0.030	0.000	0.031	0.045
.	0.000	0.198	0.323	0.129	0.851	0.146	.
1981:3 to 1990:3	0.237	-0.055	-0.040	-0.073	-0.001	0.050	0.178
.	0.014	0.131	0.380	0.078	0.760	0.258	.
1990:4 to 2019:4	0.245	-0.020	-0.026	-0.026	0.000	-0.022	0.016
.	0.000	0.341	0.319	0.250	0.843	0.444	.
1990:4 to 2005:4	0.198	-0.036	-0.052	-0.040	0.001	-0.022	0.132
.	0.001	0.167	0.070	0.117	0.066	0.447	.
2006:1 to 2019:4	0.136	-0.005	0.006	-0.002	0.000	0.048	0.013
	0.277	0.882	0.902	0.951	0.647	0.665	.

For each sample period, the first row gives the parameter estimates, and the second row the p-value of the null that the coefficient equals zero, using heteroscedasticity-consistent standard errors.

Table 4: Analysis of Rounding of CPI and UR Forecasts: Rank Correlation Tests across Individuals

CPI current	CPI next year	UR current	UR next year
A. Rounding and Accuracy			
0.068	0.120	0.043	0.160
0.768	0.905	0.677	0.959
B. Uncertainty and Rounding			
-0.064	0.098	-0.032	0.032
0.246	0.856	0.363	0.633
C. Accuracy and Uncertainty			
0.126	0.019	0.004	0.043
0.915	0.580	0.517	0.681
D. Uncertainty/Accuracy and Uncertainty			
-0.063	0.056	0.009	-0.014
0.249	0.729	0.538	0.441

Accuracy is measured by MSFE, and uncertainty by the variance of the respondents' current or next year output growth or inflation histograms. Both accuracy and uncertainty are normalized by the cross-sectional averages. Rounding is calculated as the proportion of the respondents' forecasts which are an exact multiple of 0.5.

Table 5: Reported probabilities of decline, $p^h_{j,t}$, surveys 1981:3 to 2019:4

Forecast	Current quarter	1-quarter	2-quarter	3-quarter	4-quarter
0	0.212	0.096	0.058	0.054	0.057
5	0.168	0.153	0.130	0.111	0.118
10	0.144	0.190	0.202	0.199	0.179
15	0.054	0.091	0.127	0.128	0.112
20	0.061	0.100	0.136	0.153	0.156
25	0.033	0.054	0.063	0.085	0.095
30	0.030	0.046	0.062	0.066	0.069
35	0.013	0.015	0.018	0.024	0.026
40	0.027	0.029	0.028	0.024	0.030
45	0.009	0.008	0.005	0.006	0.007
50	0.023	0.030	0.021	0.017	0.022
55	0.002	0.003	0.004	0.003	0.001
60	0.009	0.010	0.008	0.004	0.004
65	0.003	0.004	0.001	0.000	0.000
70	0.007	0.006	0.002	0.001	0.001
75	0.008	0.008	0.003	0.001	0.001
80	0.010	0.006	0.002	0.001	0.001
85	0.003	0.002	0.000	0.000	0.000
90	0.013	0.004	0.000	0.001	0.000
95	0.006	0.001	0.000	0.000	0.000
100	0.019	0.004	0.000	0.000	0.000
Proportion reported as a multiple of :					
10 or 5	0.852	0.862	0.870	0.878	0.884
10	0.555	0.522	0.520	0.519	0.522
10 or 5 (excl. 0)	0.640	0.766	0.813	0.824	0.827
No. Forecasts	4990	5067	5078	5075	5045

Notes. The table reports the proportion of probability of decline forecasts ($p^h_{j,t}$) reported as the value given in the first column, for $h = 0, 1, \ldots, 4$.

Table 6: Probability of Decline Forecasts: Rank Correlation Tests across Individuals

Rounding to 10 and Accuracy			
Rounding includes zero		Rounding excludes zero	
$h = 0$	$h = 4$	$h = 0$	$h = 4$
-0.065	0.154	0.596	0.361
0.241	0.953	1.000	1.000
Rounding to 10 and Uncertainty			
Rounding includes zero		Rounding excludes zero	
$h = 0$	$h = 4$	$h = 0$	$h = 4$
-0.248	-0.127	0.083	-0.020
0.003	0.084	0.816	0.414
Rounding to 5 and Accuracy			
Rounding includes zero		Rounding excludes zero	
$h = 0$	$h = 4$	$h = 0$	$h = 4$
0.138	0.116	0.561	0.317
0.934	0.896	1.000	1.000
Rounding to 5 and Uncertainty			
Rounding includes zero		Rounding excludes zero	
$h = 0$	$h = 4$	$h = 0$	$h = 4$
-0.073	-0.058	0.187	0.118
0.215	0.266	0.980	0.899

Accuracy is measured by normalized QPS. Uncertainty is measured by the variance of the respondents' current year output growth histograms, normalized by the cross-sectional average.

Table 7: Summary of individual OLS regressions of normalised QPS score on a dummy for rounding of current-quarter and four-quarter ahead probability of decline forecasts, assuming zero-forecasts are not rounded.

Current-quarter		Four-quarter		Current QPS	$h = 4$ QPS
Dummy	p-value	Dummy	p-value	$\hat{p}/p$	$\hat{p}/p$
1	2	3	4	5	6
0.944	22	0.275	10	27	20
0.904	27	0.487	16	16	14

Rounding is defined as a multiple of 10, but excluding forecasts of zero.
For columns 1 and 3, the table reports the cross-sectional mean and standard deviation of the individual coefficient estimates, for columns 2 and 4 the number of p-values less than 0.05 and 0.10. For columns 5 and 6 we calculate the ratio of the QPS score for $\hat{p}$ to the QPS score for p for each individual. The table entries are the number of times the ratio was less than 1 (favouring $\hat{p}$, first row) and the number of times it was less than 0.9 (second row).
The results in the columns 5 and 6 are based on a triangular kernel and Silverman's 'rule of thumb' (see footnote in main text). If instead a Gaussian kernel is used the entries in column 5 change to 31 and 15, and those in column 6 are unchanged.
The table summarises the results of the separate regressions for the 36 respondents who made 40 or more forecasts.

Table 8: Simulation results. The effects of rounding on event forecasting, on comparisons of true and rounded forecasts, and correcting rounded forecasts

			Forecasting Falls			Forecasting p^0 with $\hat{p}$		
Hetero.	σ_{ε_i}	$E\left(p^0 - 1_{w<0}\right)^2$	R_1	R_2	R_3	R_1	R_2	R_3
			Noisy Information					
0	1	0.114	1.003	1.001	1.003	0.156	0.084	0.156
0	1	0.111	1.000	1.000	1.000	0.148	0.072	0.148
0	1	0.118	1.005	1.004	1.005	0.167	0.094	0.167
1	1	0.140	1.002	1.001	1.002	0.701	1.223	0.859
1	1	0.114	1.000	0.999	1.000	0.131	0.118	0.135
1	1	0.151	1.005	1.003	1.005	4.058	7.174	6.189
0	3	0.152	1.002	1.001	1.002	0.274	0.155	0.274
0	3	0.150	1.000	0.999	1.000	0.263	0.145	0.263
0	3	0.154	1.004	1.003	1.004	0.288	0.169	0.288
1	3	0.157	1.002	1.001	1.002	0.331	0.210	0.329
1	3	0.151	0.999	0.999	0.999	0.304	0.160	0.300
1	3	0.159	1.005	1.003	1.005	0.475	0.507	0.502
			Private Information					
0	1	0.112	1.003	1.001	1.003	0.151	0.083	0.151
0	1	0.109	1.000	0.999	1.000	0.142	0.072	0.142
0	1	0.115	1.005	1.004	1.005	0.158	0.091	0.158
1	1	0.134	1.002	1.001	1.002	0.536	0.882	0.633
1	1	0.112	1.000	0.999	1.000	0.126	0.094	0.132
1	1	0.144	1.006	1.004	1.006	3.110	5.420	4.454
0	3	0.143	1.003	1.001	1.003	0.239	0.133	0.239
0	3	0.142	1.000	0.999	1.000	0.228	0.124	0.228
0	3	0.145	1.005	1.003	1.005	0.250	0.146	0.250
1	3	0.147	1.003	1.001	1.002	0.261	0.165	0.259
1	3	0.144	1.001	0.999	1.001	0.240	0.135	0.237
1	3	0.149	1.004	1.003	1.004	0.350	0.339	0.365
			Diagnostic Expectations					
0	1	0.147	1.003	1.004	1.003	0.079	0.118	0.079
0	1	0.142	1.001	1.001	1.001	0.074	0.109	0.074
0	1	0.153	1.005	1.007	1.005	0.084	0.129	0.084
1	1	0.208	1.003	1.004	1.003	0.576	0.626	0.710
1	1	0.147	1.000	1.000	1.000	0.049	0.073	0.051
1	1	0.260	1.005	1.006	1.005	3.849	4.187	5.852
0	3	0.259	1.003	1.005	1.003	0.072	0.124	0.072
0	3	0.249	1.002	1.003	1.002	0.068	0.109	0.068
0	3	0.268	1.005	1.007	1.005	0.079	0.137	0.079
1	3	0.342	1.003	1.005	1.003	0.083	0.125	0.084
1	3	0.258	1.001	1.003	1.001	0.047	0.092	0.046
1	3	0.395	1.005	1.007	1.004	0.276	0.300	0.293

In each set of 3 rows, the first is the mean across respondents, and the second and third rows are the minimum and maximum across respondents.
A 0 in the first column indicates agents are homogeneous, and a 1 indicates heterogeneity.
R_1, denotes rounding to a multiple of 0.1 with probability 0.4. R_2 makes the probability of rounding (to a multiple of 0.1) equal to p^0. R_3 sets the probability of rounding (to a multiple of 0.1) to (0.4 times) the ratio of the agent's forecast-error variance to the median.
The results for 'Forecasting Falls' are for the QPS scores for forecasting the event with a rounded probability, to the forecast using the true probability. The last three columns are the expected squared errors of the corrected forecasts to the rounded forecasts.

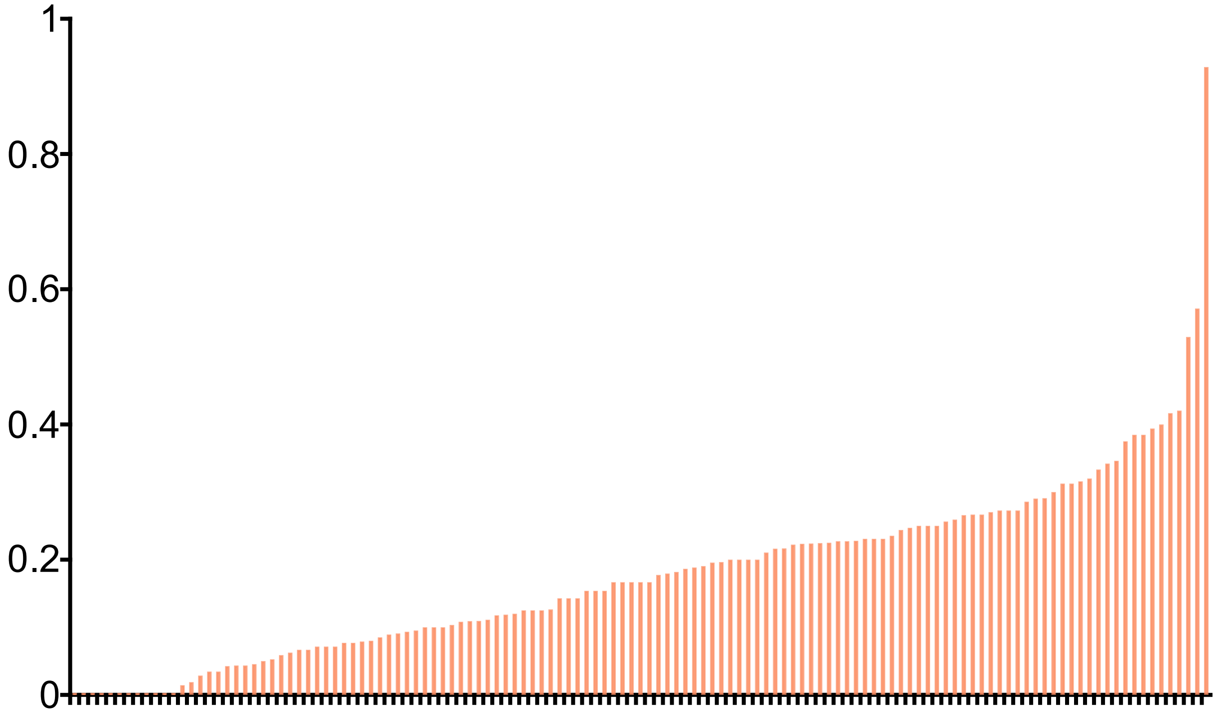

Figure 1: Proportion of each respondent's CPI forecasts (current-year) which are a multiple of 0.5

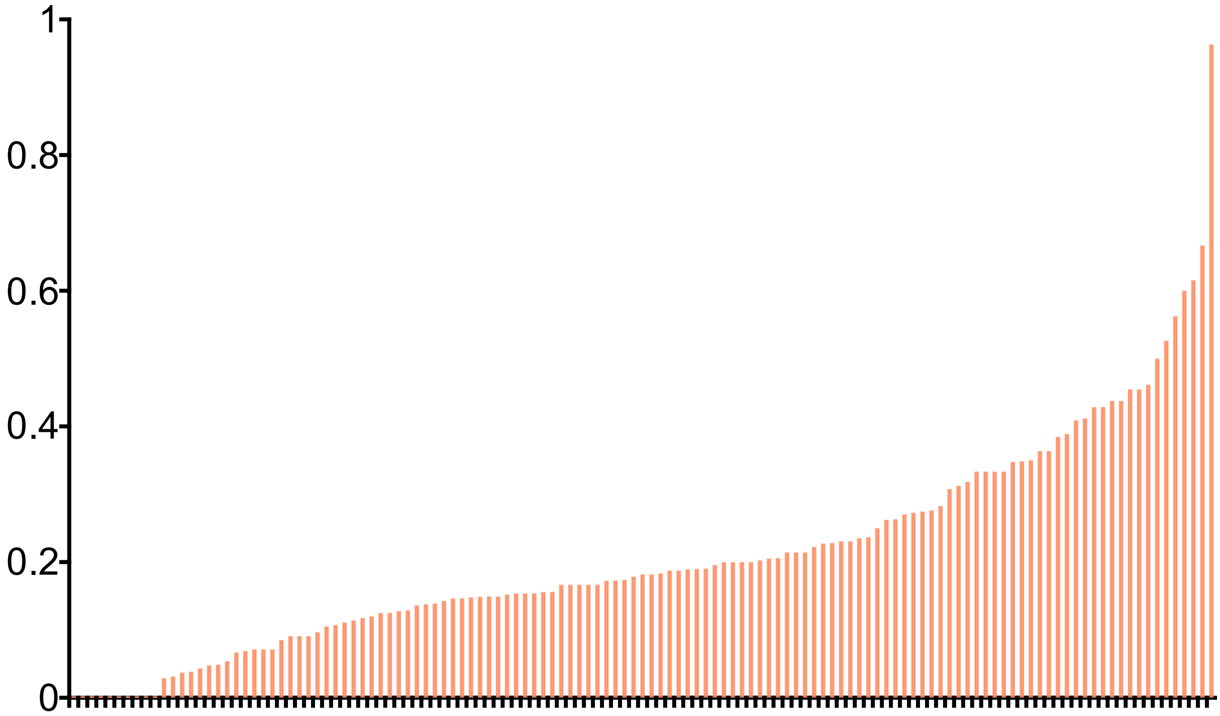

Figure 2: Proportion of each respondent's CPI forecasts (next-year) which are a multiple of 0.5

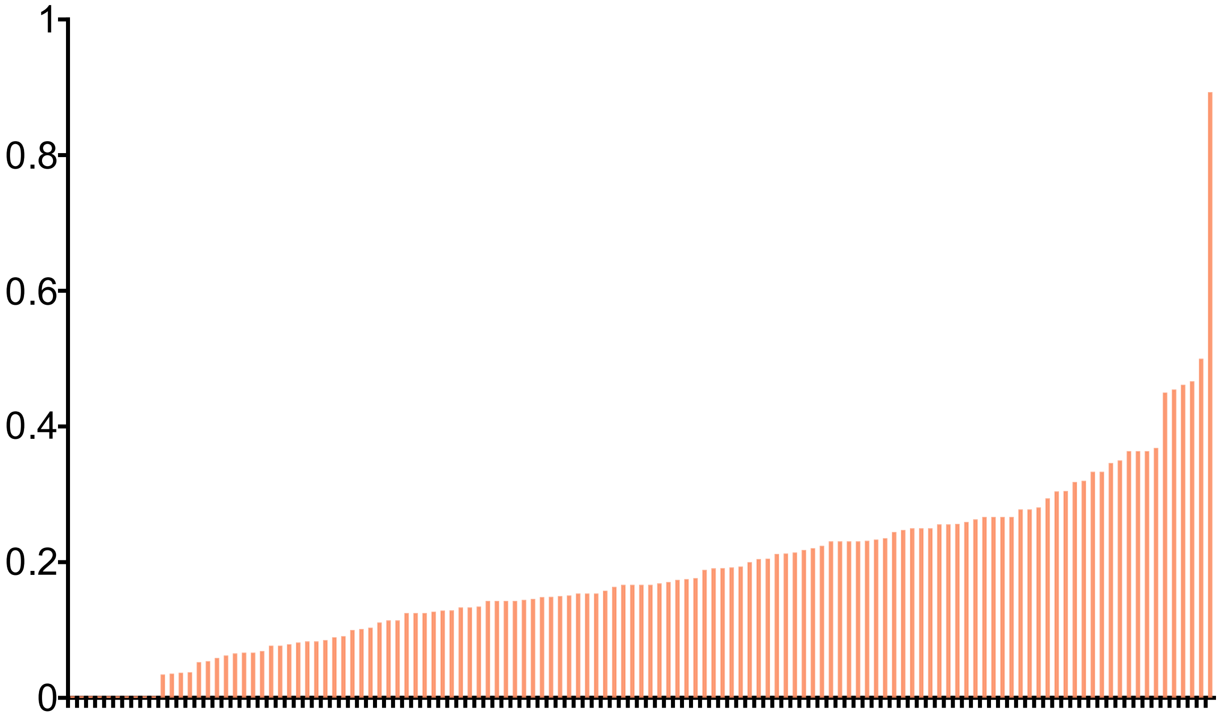

Figure 3: Proportion of each respondent's UR forecasts (current-year) which are a multiple of 0.5

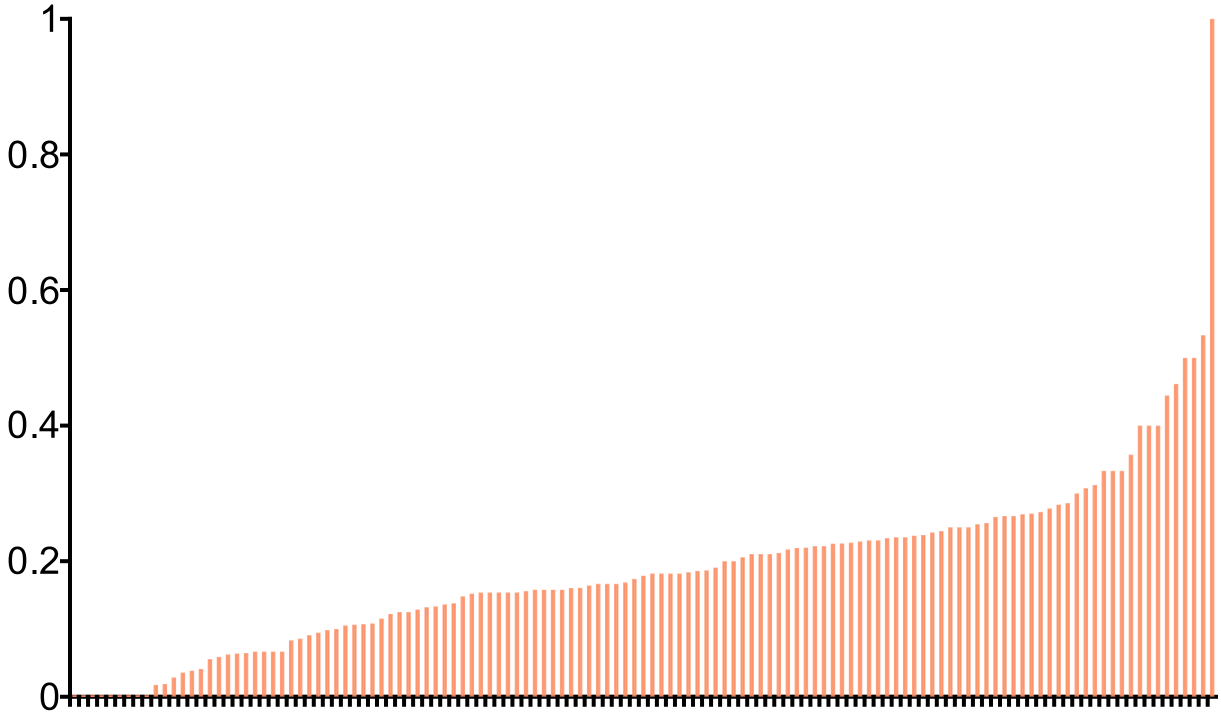

Figure 4: Proportion of each respondent's UR forecasts (next-year) which are a multiple of 0.5

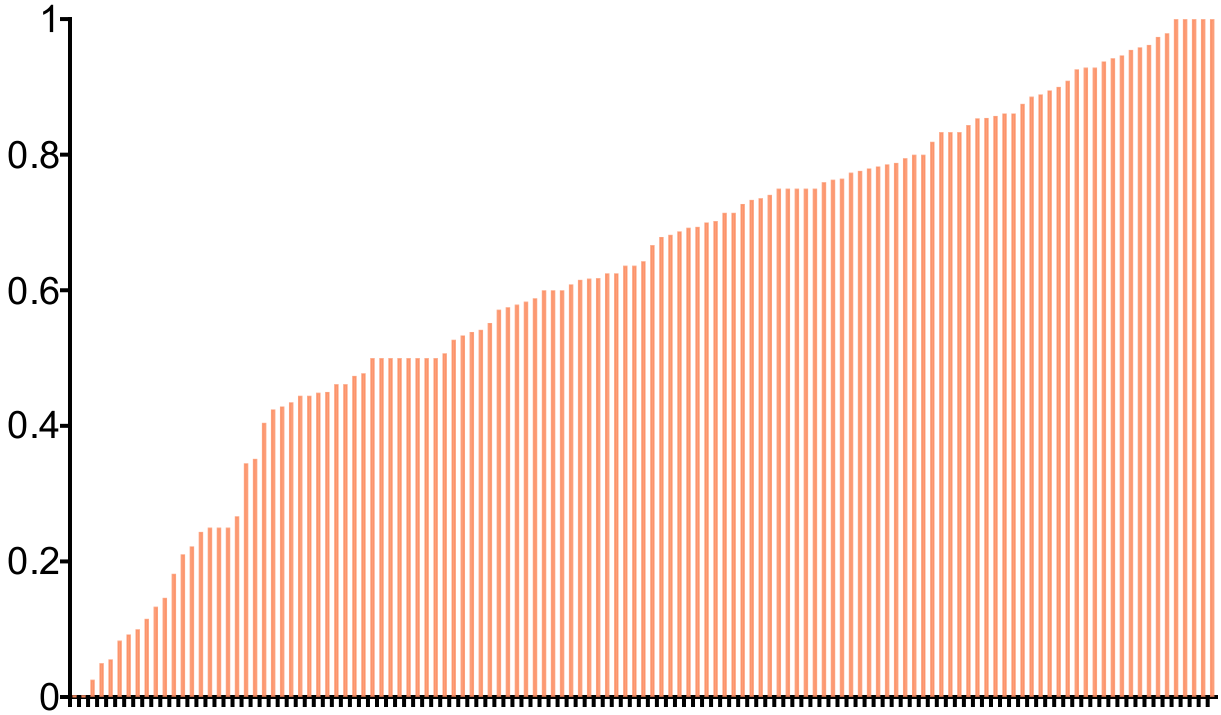

Figure 5: Proportion of each respondent's current-quarter probabilities of decline which are rounded to 10, excluding zeros

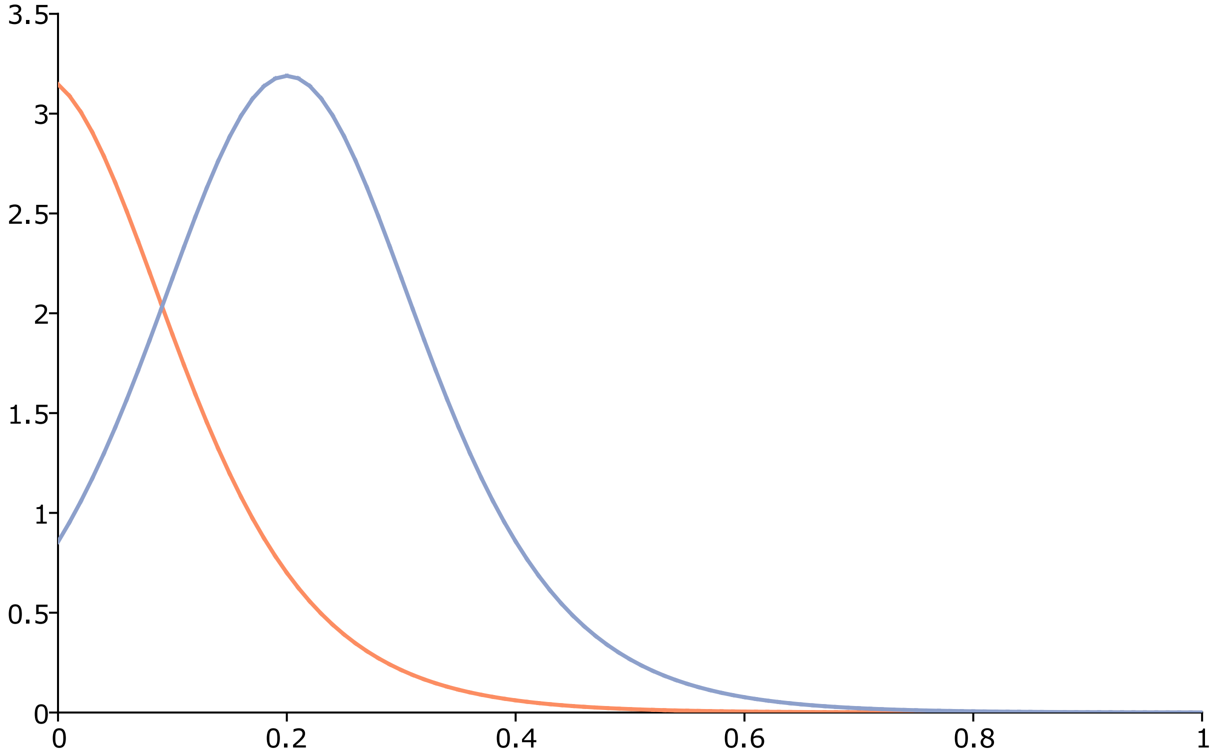

Figure 6: ϕ^l (left) and ϕ^h (right) densities in the model of the cross-sectional distribution of the current-quarter reported probabilities of decline (zeros assumed to not denote rounding). The estimated location and scale parameters are $\mu_l, \sigma_l, \mu_h, \sigma_h = -0.0181, 0.0784, 0.2001, 0.1237$.

7 Appendix to section 4.3

The data generation process for quarterly output growth in the simulations is an $AR(1)$:

$$y_t = \beta_0 + \beta y_{t-1} + \eta_t$$

where η_t is an iid gaussian innovation, $\eta_t \sim N\left(0, \sigma_\eta^2\right)$. Individual forecasters each receive a signal s_{it}:

$$s_{it} = y_t + \varepsilon_{it},$$

where $\varepsilon_{it} \sim N\left(0, \sigma_{\varepsilon_i}^2\right)$, and $\sigma_{\varepsilon_i}^2 = \sigma_\varepsilon^2\ \forall_i$ signifies all agents receive the same quality signal, and are homogeneous.

In the base case of noisy information NI, agent i's information set at time t, $\mathcal{I}_{i,t} = \{s_{it}, s_{it,t-1}, \ldots\}$, the history of signals received by agent i through t. Past values of y are not observed. Under private information PI, $\mathcal{I}_{i,t}$ is augmented with y_{t-1}, $y_{t-2}, \ldots$.

Noisy Information (NI) and Diagnostic Expectations (DE)

The forecast of t that incorporates s_{it} is given by:

$$\begin{aligned} f_{it|t} &= K_i s_{it} + (1 - K_i) f_{it|t-1} &(5)\\ &= f_{it|t-1} + K_i \left(s_{it} - f_{it|t-1}\right). &(6)\end{aligned}$$

This updates the forecast of t based on information through $t-1$, $f_{it|t-1}$, optimally in a MMSE-sense. The optimal weight is given by $K_i = \Sigma_i / \left(\Sigma_i + \sigma_{\varepsilon_i}^2\right)$, where:

$$\Sigma_i = \frac{1}{2}\left(-(1-\beta)^2 \sigma_{\varepsilon_i}^2 + \sigma_\eta^2 + \sqrt{\left[(1-\beta)^2 \sigma_{\varepsilon_i}^2 - \sigma_\eta^2\right]^2 + 4\sigma_{\varepsilon_i}^2 \sigma_\eta^2}\right)$$

(see,e.g., Bordalo *et al.* (2020)). The 1-step forecast is calculated from (11), as $f_{i,t+1|t} = \beta_0 + \beta f_{it|t}$.

The steady-state variance of the forecast error is:

$$Var\left(y_t - f_{it|t} | \mathcal{I}_{i,t}\right) = \frac{\Sigma_i \sigma_{\varepsilon_i}^2}{\Sigma_i + \sigma_{\varepsilon_i}^2}$$

and:

$$Var\left(y_{t+1} - f_{it+1t} | \mathcal{I}_{i,t}\right) = \frac{\beta^2 \sigma_{\varepsilon_i}^2 \Sigma_i}{\sigma_{\varepsilon_i}^2 + \Sigma_i} + \sigma_\eta^2.$$

Under Diagnostic Expectations, (6) becomes:

$$f_{it|t} = f_{it|t-1} + (1+\theta) K_i \left(s_{it} - f_{it|t-1}\right). \tag{7}$$

where $\theta > 0$ indicates news is overweighted relative to the optimal amount given by the Kalman gain K.

Private Information (PI)

The rational expectations forecast for agent i under PI is given by:

$$
\begin{aligned}
f_{it|t} &= \lambda_i s_{it} + (1-\lambda_i)(\beta_0 + \beta y_{t-1}) && (8)\\
&= \beta_0 + \beta y_{t-1} + \lambda_i [s_{it} - (\beta_0 - \beta y_{t-1})] && (9)\\
&= \beta_0 + \beta y_{t-1} + \lambda_i \eta_t + \lambda \varepsilon_{it} && (10)
\end{aligned}
$$

where $\lambda_i = \sigma_\eta^2 \left(\sigma_\eta^2 + \sigma_{\varepsilon_i}^2\right)^{-1}$. That is, it optimally combines the model forecast of t based on information up to $t-1$, $\beta_0 + \beta y_{t-1}$, with the time t signal s_{it}. The 1-step forecast of $t+1$ is simply, as above:

$$f_{i,t+1|t} = \beta_0 + \beta f_{it|t} \quad (11)$$

and for h-steps ahead:

$$f_{i,t+h|t} = \frac{\beta_0 \left(1-\beta^h\right)}{1-\beta} + \beta^h f_{it|t}.$$

The variances of the forecast errors are:

$$
\begin{aligned}
Var\left(y_t - f_{it|t} | \mathcal{I}_{i,t}\right) &= \frac{\sigma_{\varepsilon_i}^2 \sigma_\eta^2}{\sigma_{\varepsilon_i}^2 + \sigma_\eta^2} && (12)\\
Var\left(y_{t+1} - f_{it+!|t} | \mathcal{I}_{i,t}\right) &= \frac{\beta^2 \sigma_{\varepsilon_i}^2 \sigma_\eta^2}{\sigma_{\varepsilon_i}^2 + \sigma_\eta^2} + \sigma_\eta^2.
\end{aligned}
$$

If we let f_{it} and σ_{it}^2 denote the forecast, and forecast-error variance for one of NI, DE and PI, for a given horizon, then forecast probabilities of decline are given by:

$$p_{it} = \Phi\left(\frac{-f_{it}}{\sigma_{it}}\right). \quad (13)$$

Calibration

The model is loosely calibrated on U.S. real quarterly GDP growth We suppose $\beta_0 = 0.50$, and $\beta = 0.36$. This reproduces the unconditional growth rate of quarterly real GDP of 0.78 for the period 1947:1 – 2018:2 (2018:3 data vintage). The AR(1) model estimated standard error is $\sigma_\eta = 0.88$. These values are used for β_0, β and σ_η throughout.

In the private information model, setting $\sigma_\varepsilon = 3$ approximately reproduces the (average over time) cross-sectional standard deviation in the current-quarter output growth forecasts (1992 – 2018) of 0.22 assuming homogeneous forecasters. Disagreement varies inversely with σ_ε, because higher σ_ε means less weight is given to private signals, which are the only source of disagreement. However, setting $\sigma_\varepsilon = 3$ when $\sigma_\eta = 0.88$ suggests a weight of less than one tenth on the signal, and we also experiment with $\sigma_\varepsilon = 1$.

Forecast heterogeneity is determined such that the standard deviations of the signals are evenly spaced from 1 to 3, when the signals are (relatively) informative, $\sigma_\varepsilon = 1$, and from 3 to 9 when $\sigma_\varepsilon = 3$.

For DE, we set $\theta = \frac{1}{2}$ - all other quantities under DE, such as the forecast-error variance, are unchanged relative to NI (see Bordalo *et al.* (2020)).