Chess endgames: data and strategy

CHESS ENDGAMES: DATA AND STRATEGY

J.A. Tamplin, G.M^c^C. Haworth
jat@jaet.org; guy_haworth@hotmail.com, http:// http://www.jaet.org/jat/

Abstract While Nalimov's endgame tables for Western Chess are the most used today, their Depth-to-Mate metric is not the only one and not the most effective in use. The authors have developed and used new programs to create tables to alternative metrics and recommend better strategies for endgame play.

Keywords: chess, conversion, data, depth, endgame, goal, move count, statistics, strategy

1. Introduction

Chess endgames tables (EGTs) to the 'DTM' Depth to Mate metric are the most commonly used, thanks to codes and production work by Nalimov (Nalimov, Haworth, and Heinz, 2000a,b; Hyatt, 2000). DTM data is of interest in itself, even if *conversion*, i.e., change of force, is usually adopted as an interim objective in human play. However, more effective endgame strategies using different metrics can be adopted, particularly by computers (Haworth, 2000, 2001). A further practical disadvantage of the DTM EGTs is that, with more men, DTM increases and file-compression becomes less effective.

Here, we focus on metrics DTC, DTZ[1] and DTZ_{50}[2]; the first two were previously used by Thompson (1986, 2000) and Wirth (1999). New programs by Tamplin (2001) and Bourzutschky (2003) have enabled a complete suite of 3-to-5-man $DTC/Z/Z_{50}$ EGTs to be produced.

Section 2 outlines these two new algorithms. Sections 3 to 5 review the new DTC, DTZ and DTZ_{50} data tabled in the Appendix. Finally, improved endgame strategies are recommended for the 50-move context.

[1] DTC ≡ *Depth to Conversion*, i.e., to force change and/or mate.
DTZ ≡ *Depth to (Move-Count) Zeroing (Move)*, i.e., to P-push, force change and/or mate.
[2] DTZ_k ≡ DTZ, but *draw* if the 'win' can be pre-empted by a *k*-move draw claim.

2. New Approaches to EGT Generation

Below we briefly describe two new approaches to EGT generation. The first one is described adequately in the literature; the second so far not.

2.1 Tamplin's Wu-Beal Code

Tamplin (2001) combined the Wu-Beal (2001a,b) algorithm with Nalimov indexing in a new code whose objectives were primarily Nalimov-compatibility, simplicity, maintainability and portability. Most pawnless 3-to-5-man DTC EGTs were generated, the new code including an inverse-index function mirroring Nalimov's index function.

2.2 Bourzutschky's Modified-Nalimov Code

Bourzutschky (2003) modified Nalimov's DTM-code to enable it also to generate EGTs to metrics DTC_k and DTZ_k. This involved generalising some DTM-specific aspects of the algorithm, as well as the obvious changes to the iterative formula for deriving depth. For DTC, the code retains the efficiencies of the DTM-code while requiring maxDTC rather than maxDTM cycles. Because EGT generation to the DTZ metric has not yet been implemented generically as a sequence of sub-EGT generations, each based on a fixed pawn structure, this is not the case for DTZ_k computations. These can also require somewhat more than DTC cycles but the difference is insignificant.

3. The DTC Data

DTC EGTs are interesting, not only for completeness, but because *conversion* is an intuitively obvious objective and the DTC EGTs document precisely the phase of play when the material nominated is on the board.

The remaining 3-to-5-man DTC EGTs were generated. Table 1 in the Appendix lists for each endgame the number of positions of maxDTC, wtm/btm and 1-0/0-1. The ICGA (2003) website provides further data, including %-wins, illustrative maxDTC positions and DTC-minimaxing lines. Because there are many wins in 1, the *% of positions won* does not characterise well the presence of wins in an endgame. Similarly, maxDTC is not a good indicator. We therefore suggest a new characteristic,

$$\textit{Win-Presence} \equiv \%\_\text{of\_positions\_won} \times (\text{Average DTC of Win})$$

This is not unduly affected by the usual peak of wins in 1 or by the long tail of deep wins, and is in fact related to the number of moves for which a win is present on the board.

3.1 A Review of the DTC Data

A first housekeeping point to be made is that this data often differs from Wirth's data (Wirth and Nievergelt, 1999; Tamplin, 2003). The explanation is simple. First, Wirth has exactly one representative of each equivalence class of positions, including the harder case of both Kings being on a1-h8. Nalimov would count {wKc3Qb3(c2)/bKa1} as two positions rather than one.

Second, Wirth's code, based on the inherited RETROENGINE, assumes that all conversions are effected by the winner. This is not so: the loser is sometimes forced to convert to loss, e.g., {wKe1Qb1Rf1/bKa1}, in which case Wirth's depth is too great by one.

Tamplin's (2003) and Bourzutschky's (2003) codes both measure depth consistently in *winner's moves*. Also, they do not allow 'realistic' but voluntary conversions, e.g., {wKe1Qf1Rb1/bKa1}, by the loser, a feature of Thompson's original DTC EGT code (Thompson, 1986) which chose to move to the position with greatest DTC even if a capture was involved.

The sub-6-man compressed DTC EGTs are 62.1% the size of the DTM EGTs, usefully saving 2.8GB disc space.

The maxDTC=114 wins in KNNKP and KQPKQ are already known. KBNK wtm scores the highest in *Win-Presence* terms: maxDTC = 33, average DTC = 24.68 and 99.51% of positions are 1-0 wins.

4. The DTZ Data

The DTZ metric is necessary if the length of the current phase of play is to be *guarded* in the context of chess' *k*-move rule, *k* currently being 50. It was used pragmatically by Thompson (1986) to compute the KQPKQ and KRPKR EGTs when RAM was relatively scarce.

Bourzutschky (2003) generated some DTZ EGTs where maxDTZ > 50 and Tamplin (2003) completed the sub-6-man DTZ EGT suite. The computation continues to be a major feat as it cannot currently use Nalimov's bitvector-based algorithm which reduces RAM requirements by a factor of 4 to 16.

Table 2 in the Appendix lists the results which differ from the DTC data. KNNKP with maxDTZ = 82 features the deepest endings. DTZ EGTs are commendably compact relative to DTM and DTC EGTs. The KPPPK wtm DTZ EGT is an extreme example, being only 2% the size of the DTM EGT. In total, the sub-6-man compressed DTZ EGTs are 52.9% the size of the DTM EGTs, usefully saving some 3.5GB of disc space.

5. The DTZ_{50} Data

Bourzutschky (2003) and Tamplin (2003) also generated DTZ_{50} EGTs, not only for those cases where maxDTZ > 50, but for endgames directly or indirectly dependent on these as illustrated in Figure 1. The DTZ_{50} metric rates as wins only those positions winnable against best play given the 50-move rule. In Figure 1, endgames for which EZ and EZ_{50} are potentially but not actually different are in brackets, and dotted lines indicate that no 50-move impact emanates from or feeds back to them.

The sub-6-man compressed DTZ_{50} EGTs are 49.8% the size of the DTM EGTs. Table 3 in the Appendix lists 3-to-5-man DTZ_{50} EGT data for endgames where $DTZ_{50} \neq$ DTZ and Table 7 gives examples of positions affected. Table 6 summarises 50-move impact, minimal for KNPKQ, considerable for KBBKN and KNNKP.

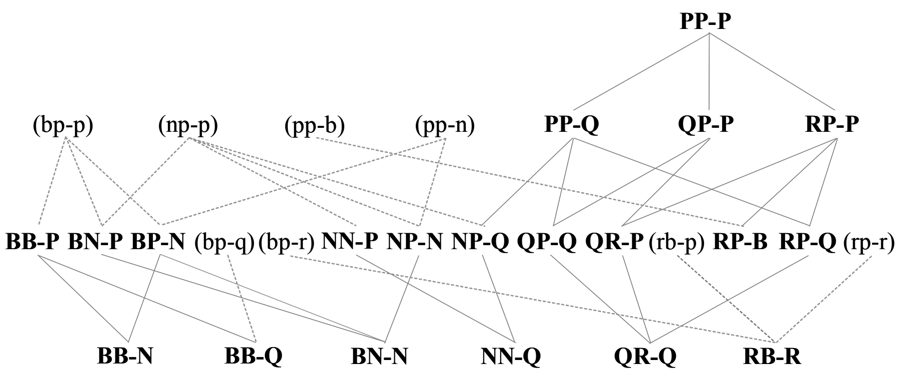

Figure 1. Endgames with $EZ_{50} \neq$ EZ.

If *KwKb* is an endgame with wtm and btm 1-0 wins impacted by the 50-move rule, *KwxKb* and *KwKby* are also impacted by the rule. This observation, coupled with Thompson's DTC results (Tamplin and Haworth, 2001) and the DTM results of Nalimov (Hyatt, 2000) and Bourzutschky (2003) indicate that many 6-man endgames are affected. Tamplin (2003) has computed some of these 6-man endgames' EGTs to the DTZ and DTZ_{50} metrics.

In contrast with KNNKP, KBBKNN has the majority of its wins frustrated, and few wins can be retained by deeper strategy in the current phase. There are significant percentages of frustrated 0-1 wins in KBBBKQ, and of delayed 1-0 wins in KBBBKN and KBBNKN.

Elsewhere, there is only the merest hint of the 50-move impact that might follow and we would expect that hint to become fainter as the number of men increases.

6. Endgame Strategies

Let *dtx* be the depth by, and E*x* an EGT to, the metric DT*x*. Let Sx^- be an endgame strategy minimising *dtx*, e.g., SZ^-, or SZ_{50}^-, and let Sx^+ be a strategy maximising *dtx*. Further, let SZº be an endgame strategy *guarding* the length of the current phase in the context of a *k*-move rule and a remaining *mleft* moves before a possible draw claim. By definition, if *dtx* > *mleft*, Sxº ≡ Sx^-.

Let $Ss_1s_2s_3$ be an endgame strategy using strategies Ss_1, Ss_2, and Ss_3 in turn to subset the choice of moves, e.g., $SZ^oZ_{50}^-M^-Z^-$ which safeguards current phase length and 50-move wins, and then minimises *dtm* and *dtz* in turn.

As conjectured by Haworth (2000), KQPKQ and KBBKNN provide positions where all combinations of SC^-, SM^- and SZ^- *fail* to safeguard a win available under the 50-move rule: the examples here were found by Bourzutschky (2003). Similar positions for other endgames were found by Tamplin (2003). Some strategy-driven lines are listed in Appendix 1 after Table 5.

6.1 New Endgame Strategies

SZ_{50}^- wins any game winnable against best play under the 50-move drawing rule. Here, we suggest ways to finesse wins against fallible opposition. If the current phase of play is not unavoidably overlong, strategy $SZ^oZ_{50}^-Z^-$, effectively $SZ^oZ^- \equiv SZ^-$, completes it without a draw claim.

For positions where DTZ_{50} indicates *draw*, the table EZ_{50} can be supplemented by the position's DTR[3] value. Let this hybrid table be EH_{50}, implicitly defining metric DTH_{50}. Note that EZ_{50} is visible within EH_{50}. Since the intention is to use EH_{50} only in conjunction with EZ, let the table $E\delta(H/Z)_{50}Z \equiv \{\delta(DT(H/Z)_{50}, DTZ)\}$, giving a compact encoding[4] of $E(H/Z)_{50}$ decodable with the use of EZ. With $E\delta Z_{50}Z = \Phi$ if $EZ_{50} \equiv EZ$, sub-6-man compressed $E\delta Z_{50}Z$ EGTs are only 0.7% the size of the DTM EGTs.

The strategy $SZ^oH_{50}^-$ guards the length of the current phase, wins all games which are wins under the 50-move rule, and minimaxes DTR, but only tactically, when the 50-move rule intervenes.

In position NN-P3, $SZ^oH_{50}^-$ makes the optimal move-choice[5]. In contrast, $SZ^oZ_{50}^-$ can, and Sσ (σ ≡ C^-, M^-, Z^-, $Z^oZ_{50}^-Z^-$) does, concede DTR depth. However, $SZ^oH_{50}^-$ has two flaws, the first being a major one. It can draw by repeating positions, e.g., position NN-P4[6]. $SZ^oH_{50}^-$ should therefore be augmented by as deep and perceptive a forward search as possible, denoted here by '*' as in $SZ^oH_{50}^{-*}$.

[3] DTR ≡ *Depth by The Rule* (Haworth, 2000, 2001), i.e. the minimum *k* s.t. DTZ_k is a win.
[4] We chose 0 ≡ "EZ code = Ex_k code", 1 ≡ "new EZ_{50} draw", δ+1 ≡ "0 < DTx – DTZ = δ".
[5] $SZ^oH_{50}^-$ - SH_{50}^+: 1. Nb1+' Ka4'. White retains DTR=51 and converts in 31 moves.
[6] NN-P4, $SZ^oH_{50}^-$ - SH_{50}^+: 1. Nd5+? Kc4' 2. Ndc3 Kb4' {NN-P4 repeated}.

If position NN-P4, with dtz_{51} = 25, has just 25 moves left in the phase, it also shows $SZ^oH_{50}^-$ failing to achieve minimal DTR. The move Nd5+ is optimal for SH_{50}^- but DTZ_{51}-suboptimal, a fact not visible in the EGT EH_{50}. After Nd5+, SZ^o limits the move choice and puts a DTR of 51 out of reach. Again, forward search helps, this time aiming to control DTR.

Any strategy can be sharpened by the opponent sensitivity of an adaptive, opponent model (Haworth, 2003; Haworth and Andrist, 2003).

7. EGT Integrity

All EGT files were given md5sum signatures to guard against subsequent corruption. The EGTs were checked for errors in various ways.

- DT*x* EGTs {E*x*}, *x* = C, Z and Z_{50}, verified by Nalimov's standard test.
- consistency of the {E(C/M/Z)} EGTs confirmed
  theoretical values found identical with $dtm \geq dtc \geq dtz$.
- DTC EGT statistics were also found compatible with those of Wirth.
- consistency of the $\{EZ_{50}\}$ and {EZ} EGTs confirmed
  linear checks confirm $EZ_{50} \equiv EZ$ except for known subset,
  values identical with $dtz_{50} \geq dtz$, or 'EZ' win/loss an 'EZ_{50}' draw.

8. Summary

This paper records the separate initiatives of Tamplin (2003) and Bourzutschky (2003) in creating new codes capable of generating non-DTM EGTs. It also reviews the new DTC/Z/Z_{50} data produced by the combination of these codes. The DTC, DTZ and DTZ_{50} EGTs (EC, EZ and EZ_{50}) are increasingly compact compared to the DTM EGTs, an incidental but practical benefit with 3-to-6-man DTM EGTs estimated to be 1 to 2 TB in size.

Together, the sub-6-man compressed EZ and $E\delta Z_{50}Z$ EGTs are 53.6% the size of the EM EGTs. To date, the equivalent 6-man EGTs are 63.8% the size of their EM EGT counterparts but these do not yet involve Pawns

Although the computation of DTR data remains a future challenge, table EZ_{50} may in principle be augmented by DTR values where $dtr > 50$ to give table EH_{50}. This table may be used to minimise dtz_{50} when $dtz_{50} \leq 50$, and to minimax *dtr* with the assistance of forward-search when $dtr \geq 50$.

Clearly, there are more effective and efficient endgame strategies than the commonly used SM^-. It is recommended that SZ^oM^-, $SZ^oZ_{50}^-Z^{-(*)}$, $SZ^oZ_{50}^-Z^-H_{50}^{-(*)}$, $SZ^oH_{50}^{-*}$ and perhaps other strategies are considered, and that the EZ, $E\delta Z_{50}Z$ and $E\delta H_{50}Z$ EGTs are made available to enable their use.

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Appendix: Chess Endgame Data and Examples

Endgame				DTC Metric: # of maximal positions				DTC Metric: max depths, moves			
				1-0		0-1		1-0		0-1	
Name	**GBR**	**#**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBK	**0010.00**	3	2-1	0	0	0	0	—	—	—	—
KNK	**0001.00**	3	2-1	0	0	0	0	—	—	—	—
KPK	**0000.10**	3	2-1	3	2	0	0	19	19	—	—
KQK	**1000.00**	3	2-1	1	8	0	0	10	10	—	—
KRK	**0100.00**	3	2-1	139	433	0	0	16	16	—	—
KBKB	**0040.00**	4	2-2	52	14	14	52	1	0	0	1
KBKN	**0013.00**	4	2-2	2	1	1	5	1	0	0	1
KBKP	**0010.01**	4	2-2	104	28	6	14	1	0	5	6
KNKN	**0004.00**	4	2-2	5	1	1	5	1	0	0	1
KNKP	**0001.01**	4	2-2	29	7	3	3	7	6	12	13
KPKP	**0000.11**	4	2-2	1	1	1	1	14	14	14	14
KQKB	**1030.00**	4	2-2	980	4,837	0	0	12	12	—	—
KQKN	**1003.00**	4	2-2	5	19	0	0	19	19	—	—
KQKP	**1000.01**	4	2-2	1	1	20	20	26	26	1	2
KQKQ	**4000.00**	4	2-2	5	3	3	5	10	9	9	10
KQKR	**1300.00**	4	2-2	2	11	55	291	31	31	2	3
KRKB	**0130.00**	4	2-2	29	1	0	0	18	18	—	—
KRKN	**0103.00**	4	2-2	2	2	1	4	27	27	0	1
KRKP	**0100.01**	4	2-2	28	42	3	3	16	16	10	11
KRKR	**0400.00**	4	2-2	59	111	111	59	4	3	3	4
KBBK	**0020.00**	4	3-1	16	59	0	0	19	19	—	—
KBNK	**0011.00**	4	3-1	144	436	0	0	33	33	—	—
KBPK	**0010.10**	4	3-1	2	8	0	0	21	21	—	—
KNNK	**0002.00**	4	3-1	77	15	0	0	1	0	—	—
KNPK	**0001.10**	4	3-1	24	32	0	0	22	22	—	—
KPPK	**0000.20**	4	3-1	62	21	0	0	16	16	—	—
KQBK	**1010.00**	4	3-1	2,411	14,012	0	0	6	6	—	—
KQNK	**1001.00**	4	3-1	4,932	23,203	0	0	7	7	—	—
KQPK	**1000.10**	4	3-1	75	175	0	0	7	7	—	—
KQQK	**2000.00**	4	3-1	3,280	13,005	0	0	3	3	—	—
KQRK	**1100.00**	4	3-1	44	158	0	0	4	4	—	—
KRBK	**0110.00**	4	3-1	1	6	0	0	12	12	—	—
KRNK	**0101.00**	4	3-1	324	1,017	0	0	12	12	—	—
KRPK	**0100.10**	4	3-1	376	1,885	0	0	8	8	—	—
KRRK	**0200.00**	4	3-1	68	287	0	0	5	5	—	—
KBBKB	**0050.00**	5	3-2	503	6	141	546	6	6	1	2
KBBKN	**0023.00**	5	3-2	34	53	44	222	66	66	0	1
KBBKP	**0020.01**	5	3-2	34	69	5	11	21	21	8	9
KBBKQ	**3020.00**	5	3-2	248	58	74	15	4	3	71	71
KBBKR	**0320.00**	5	3-2	26	7	2	6	7	6	8	9
KBNKB	**0041.00**	5	3-2	28	19	133	514	13	12	1	2
KBNKN	**0014.00**	5	3-2	2	1	104	533	77	76	0	1
KBNKP	**0011.01**	5	3-2	1	2	523	535	26	26	8	9
KBNKQ	**3011.00**	5	3-2	79	1	22	4	5	5	42	42
KBNKR	**0311.00**	5	3-2	127	23	4	2	6	5	12	13
KBPKB	**0040.10**	5	3-2	14	14	508	1,524	40	39	2	3
KBPKN	**0013.10**	5	3-2	16	6	23	86	42	42	3	4
KBPKP	**0010.11**	5	3-2	92	52	27	23	53	53	6	7
KBPKQ	**3010.10**	5	3-2	30	30	3	2	4	3	42	42
KBPKR	**0310.10**	5	3-2	76	53	5	6	13	12	20	21

Table 1a. Chess Endgames: 3-to-5-man DTC data.

Endgame				DTC Metric							
				# of maximal positions				max depths, moves			
				1-0		0-1		1-0		0-1	
Name	**GBR**	**#**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KNNKB	**0032.00**	5	3-2	251	82	51	109	4	3	0	1
KNNKN	**0005.00**	5	3-2	38	18	56	293	7	6	0	1
KNNKP	**0002.01**	5	3-2	2	4	1	1	114	113	12	13
KNNKQ	**3002.00**	5	3-2	2,387	465	10	2	1	0	63	63
KNNKR	**0302.00**	5	3-2	2	1	6	11	3	2	10	11
KNPKB	**0031.10**	5	3-2	11	3	5	18	31	30	8	9
KNPKN	**0004.10**	5	3-2	9	2	27	132	48	48	3	4
KNPKP	**0001.11**	5	3-2	1	6	6	9	33	33	13	14
KNPKQ	**3001.10**	5	3-2	2	2	1	1	5	4	43	43
KNPKR	**0301.10**	5	3-2	8	36	7	1	18	18	42	43
KPPKB	**0030.20**	5	3-2	31	34	14	34	18	17	3	4
KPPKN	**0003.20**	5	3-2	3	5	21	12	30	29	13	14
KPPKP	**0000.21**	5	3-2	2	11	66	58	28	28	11	12
KPPKQ	**3000.20**	5	3-2	14	15	19	8	6	5	30	30
KPPKR	**0300.20**	5	3-2	1	1	2	3	25	24	25	25
KQBKB	**1040.00**	5	3-2	220	998	187	645	8	8	1	2
KQBKN	**1013.00**	5	3-2	74	343	30	153	7	7	0	1
KQBKP	**1010.01**	5	3-2	5	19	791	789	11	11	1	2
KQBKQ	**4010.00**	5	3-2	33	1	1	1	30	30	16	17
KQBKR	**1310.00**	5	3-2	1	6	8,848	52,298	19	19	1	2
KQNKB	**1031.00**	5	3-2	50	158	28	64	9	9	0	1
KQNKN	**1004.00**	5	3-2	7	39	31	166	9	9	0	1
KQNKP	**1001.01**	5	3-2	7	8	928	911	17	17	1	2
KQNKQ	**4001.00**	5	3-2	7	1	1	4	35	35	13	14
KQNKR	**1301.00**	5	3-2	1	6	15	86	22	22	2	3
KQPKB	**1030.10**	5	3-2	1,122	4,328	374	1,290	9	9	1	2
KQPKN	**1003.10**	5	3-2	1	6	3	9	10	10	1	2
KQPKP	**1000.11**	5	3-2	11,817	39,633	16	16	6	6	2	3
KQPKQ	**4000.10**	5	3-2	5	13	2	4	114	113	15	16
KQPKR	**1300.10**	5	3-2	4	20	5,177	26,128	20	20	2	3
KQQKB	**2030.00**	5	3-2	4	15	0	0	4	4	—	—
KQQKN	**2003.00**	5	3-2	287	1,411	0	0	4	4	—	—
KQQKP	**2000.01**	5	3-2	18,995	19,257	140	140	3	3	1	2
KQQKQ	**5000.00**	5	3-2	2	21	31	152	25	25	6	7
KQQKR	**2300.00**	5	3-2	2	12	2,383	16,681	14	14	1	2
KQRKB	**1130.00**	5	3-2	720	2,556	0	0	5	5	—	—
KQRKN	**1103.00**	5	3-2	234	1,149	36	149	5	5	0	1
KQRKP	**1100.01**	5	3-2	104,508	131,846	683	683	3	3	1	2
KQRKQ	**4100.00**	5	3-2	3	31	1	2	60	60	8	9
KQRKR	**1400.00**	5	3-2	10	54	8,099	56,501	15	15	1	2
KRBKB	**0140.00**	5	3-2	35	46	251	951	25	25	1	2
KRBKN	**0113.00**	5	3-2	9	35	106	481	21	21	0	1
KRBKP	**0110.01**	5	3-2	2	12	4	12	11	11	4	5
KRBKQ	**3110.00**	5	3-2	1	3	5	4	7	6	41	42
KRBKR	**0410.00**	5	3-2	28	19	3	14	59	58	3	4
KRNKB	**0131.00**	5	3-2	3	6	41	89	25	25	0	1
KRNKN	**0104.00**	5	3-2	5	18	101	468	24	24	0	1
KRNKP	**0101.01**	5	3-2	65	81	2	2	15	15	10	11
KRNKQ	**3101.00**	5	3-2	24	5	7	3	9	8	46	46
KRNKR	**0401.00**	5	3-2	1	1	1	3	33	32	4	5

Table 1b. Chess Endgames: 3-to-5-man DTC data.

Endgame				DTC Metric							
				# of maximal positions				max depths, moves			
				1-0		0-1		1-0		0-1	
Name	GBR	#	w-b	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KRPKB	**0130.10**	5	3-2	11	26	502	1,672	62	62	1	2
KRPKN	**0103.10**	5	3-2	2	7	4	12	46	46	1	2
KRPKP	**0100.11**	5	3-2	184	474	17	17	9	9	10	11
KRPKQ	**3100.10**	5	3-2	5	5	5	1	9	8	78	79
KRPKR	**0400.10**	5	3-2	33	4	23	80	60	60	6	7
KRRKB	**0230.00**	5	3-2	1	4	0	0	10	10	—	—
KRRKN	**0203.00**	5	3-2	215	687	45	184	7	7	0	1
KRRKP	**0200.01**	5	3-2	16	48	988	988	9	9	1	2
KRRKQ	**3200.00**	5	3-2	14	4	2	3	15	14	20	20
KRRKR	**0500.00**	5	3-2	3	15	6,210	43,225	25	25	1	2
KBBBK	**0090.00/30**	5	4-1	116	345	0	0	10	10	—	—
KBBNK	**0021.00**	5	4-1	783	2,066	0	0	13	13	—	—
KBBPK	**0020.10**	5	4-1	3	2	0	0	16	16	—	—
KBNNK	**0012.00**	5	4-1	22	59	0	0	13	13	—	—
KBNPK	**0011.10**	5	4-1	9	45	0	0	10	10	—	—
KBPPK	**0010.20**	5	4-1	56	46	0	0	16	16	—	—
KNNNK	**0009.00/30**	5	4-1	44	180	0	0	21	21	—	—
KNNPK	**0002.10**	5	4-1	194	296	0	0	15	15	—	—
KNPPK	**0001.20**	5	4-1	2	5	0	0	12	12	—	—
KPPPK	**0000.30**	5	4-1	11	35	0	0	11	11	—	—
KQBBK	**1020.00**	5	4-1	182	673	0	0	6	6	—	—
KQBNK	**1011.00**	5	4-1	54,680	236,453	0	0	4	4	—	—
KQBPK	**1010.10**	5	4-1	68	255	0	0	6	6	—	—
KQNNK	**1002.00**	5	4-1	182	673	0	0	7	7	—	—
KQNPK	**1001.10**	5	4-1	11,789	56,328	0	0	5	5	—	—
KQPPK	**1000.20**	5	4-1	1,264	4,476	0	0	6	6	—	—
KQQBK	**2010.00**	5	4-1	96,576	412,131	0	0	3	3	—	—
KQQNK	**2001.00**	5	4-1	13	58	0	0	4	4	—	—
KQQPK	**2000.10**	5	4-1	138	732	0	0	4	4	—	—
KQQQK	**9000.00/30**	5	4-1	1,513	6,553	0	0	3	3	—	—
KQQRK	**2100.00**	5	4-1	56,174	218,959	0	0	3	3	—	—
KQRBK	**1110.00**	5	4-1	1,198	5,865	0	0	4	4	—	—
KQRNK	**1101.00**	5	4-1	7,474	31,526	0	0	4	4	—	—
KQRPK	**1100.10**	5	4-1	3	15	0	0	5	5	—	—
KQRRK	**1200.00**	5	4-1	18	87	0	0	4	4	—	—
KRBBK	**0120.00**	5	4-1	24	126	0	0	10	10	—	—
KRBNK	**0111.00**	5	4-1	8,391	26,677	0	0	7	7	—	—
KRBPK	**0110.10**	5	4-1	1	5	0	0	8	8	—	—
KRNNK	**0102.00**	5	4-1	602	2,052	0	0	10	10	—	—
KRNPK	**0101.10**	5	4-1	579	1,436	0	0	8	8	—	—
KRPPK	**0100.20**	5	4-1	4	24	0	0	8	8	—	—
KRRBK	**0210.00**	5	4-1	4,761	17,210	0	0	5	5	—	—
KRRNK	**0201.00**	5	4-1	8,533	29,009	0	0	5	5	—	—
KRRPK	**0200.10**	5	4-1	16	56	0	0	6	6	—	—
KRRRK	**0900.00/30**	5	4-1	3,566	13,290	0	0	4	4	—	—

Table 1c. Chess Endgames: 3-to-5-man DTC data.

Endgame				DTZ Metric: # of maximal positions 1-0		0-1		max depth, moves 1-0		0-1	
	GBR	#	w-b	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KPK	**0000.10**	3	2-1	8	4	0	0	10	10	—	—
KBKP	**0010.01**	4	2-2	104	28	779	585	1	0	3	4
KNKP	**0001.01**	4	2-2	23	6	6	2	6	5	8	8
KPKP	**0000.11**	4	2-2	1	1	1	1	11	10	10	11
KQKP	**1000.01**	4	2-2	1	1	20	385,976	26	26	1	1
KRKP	**0100.01**	4	2-2	2	38	3	3	13	12	10	10
KBPK	**0010.10**	4	3-1	38	42	0	0	13	13	—	—
KNPK	**0001.10**	4	3-1	108	8	0	0	13	13	—	—
KPPK	**0000.20**	4	3-1	125	152	0	0	7	7	—	—
KQPK	**1000.10**	4	3-1	25	107	0	0	3	3	—	—
KRPK	**0100.10**	4	3-1	1,643	6,556	0	0	3	3	—	—
KBBKP	**0020.01**	5	3-2	16	16	5	47	21	21	8	8
KBNKP	**0011.01**	5	3-2	202	39	494	157	20	20	8	8
KBPKB	**0040.10**	5	3-2	13	22	508	1,524	25	25	2	3
KBPKN	**0013.10**	5	3-2	20	5	23	86	30	30	3	4
KBPKP	**0010.11**	5	3-2	9	4	24	30	37	37	5	6
KBPKQ	**3010.10**	5	3-2	1,438	30	1	2	3	3	42	42
KBPKR	**0310.10**	5	3-2	5	39	5	6	13	12	18	19
KNNKP	**0002.01**	5	3-2	18	13	1	1	82	81	11	11
KNPKB	**0031.10**	5	3-2	39	33	5	18	24	24	8	9
KNPKN	**0004.10**	5	3-2	2	25	27	132	30	29	3	4
KNPKP	**0001.11**	5	3-2	1	1	12	4	23	23	7	7
KNPKQ	**3001.10**	5	3-2	2,459	4	1	1	3	3	43	43
KNPKR	**0301.10**	5	3-2	8	36	3	9	18	18	39	40
KPPKB	**0030.20**	5	3-2	2	5	1	13	12	12	1	2
KPPKN	**0003.20**	5	3-2	3	8	45	100	14	13	6	7
KPPKP	**0000.21**	5	3-2	1	3	1	4	21	21	7	7
KPPKQ	**3000.20**	5	3-2	8	15	19	16	6	5	29	29
KPPKR	**0300.20**	5	3-2	67	83	13	14	14	14	15	15
KQBKP	**1010.01**	5	3-2	5	14	791	2,934,215	11	11	1	1
KQNKP	**1001.01**	5	3-2	7	1	928	5,722,853	17	17	1	1
KQPKB	**1030.10**	5	3-2	13,462	65,629	374	1,290	5	5	1	2
KQPKN	**1003.10**	5	3-2	26	105	3	9	6	6	1	2
KQPKP	**1000.11**	5	3-2	69	2	1,024	7,412,631	5	5	1	1
KQPKQ	**4000.10**	5	3-2	1	3	2	4	71	70	15	16
KQPKR	**1300.10**	5	3-2	3	19	5,177	26,128	17	17	2	3
KQQKP	**2000.01**	5	3-2	13,425	1,987	140	16,368	3	3	1	1
KQRKP	**1100.01**	5	3-2	76,181	2,592	683	892,287	3	3	1	1
KRBKP	**0110.01**	5	3-2	2	10	4	8	11	11	4	5
KRNKP	**0101.01**	5	3-2	19	26	2	2	15	14	10	10
KRPKB	**0130.10**	5	3-2	5	7	502	1,672	53	53	1	2
KRPKN	**0103.10**	5	3-2	8	15	4	12	31	31	1	2
KRPKP	**0100.11**	5	3-2	20	22	17	18	9	9	10	10
KRPKQ	**3100.10**	5	3-2	2	5	3	1	9	8	75	76
KRPKR	**0400.10**	5	3-2	3	4	14	43	35	35	6	7
KRRKP	**0200.01**	5	3-2	16	32	988	1,506,491	9	9	1	1
KBBPK	**0020.10**	5	4-1	5	1	0	0	12	12	—	—
KBNPK	**0011.10**	5	4-1	74	199	0	0	5	5	—	—
KBPPK	**0010.20**	5	4-1	16	32	0	0	9	9	—	—
KNNPK	**0002.10**	5	4-1	6,992	7,623	0	0	8	8	—	—

Table 2a. Chess Endgames: 3-to-5-man DTZ data.

				DTZ Metric							
Endgame				# of maximal positions				max depth, moves			
				1-0		0-1		1-0		0-1	
	GBR	**#**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KNPPK	**0001.20**	5	4-1	1	7	0	0	6	6	—	—
KPPPK	**0000.30**	5	4-1	16	64	0	0	7	7	—	—
KQBPK	**1010.10**	5	4-1	2,085	6,415	0	0	3	3	—	—
KQNPK	**1001.10**	5	4-1	958	4,181	0	0	3	3	—	—
KQPPK	**1000.20**	5	4-1	20	88	0	0	3	3	—	—
KQQPK	**2000.10**	5	4-1	29	81	0	0	3	3	—	—
KQRPK	**1100.10**	5	4-1	2,330	6,022	0	0	3	3	—	—
KRBPK	**0110.10**	5	4-1	67	114	0	0	4	4	—	—
KRNPK	**0101.10**	5	4-1	36	152	0	0	4	4	—	—
KRPPK	**0100.20**	5	4-1	270	651	0	0	3	3	—	—
KRRPK	**0200.10**	5	4-1	6,122	11,124	0	0	3	3	—	—

Table 2b. Chess Endgames: 3-to-5-man DTZ data.

				DTZ_{50} Metric							
Endgame				# of maximal positions				max depth, moves			
				1-0		0-1		1-0		0-1	
	GBR	**#**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBBKN	**0023.00**	5	3-2	347,796	485,538	44	222	50	50	0	1
KBBKP	**0020.01**	5	3-2	16	16	3	4	21	21	9	10
KBBKQ	**3020.00**	5	3-2	248	58	86,896	24,793	4	3	50	50
KBNKN	**0014.00**	5	3-2	12,123	5,857	104	533	50	50	0	1
KBNKP	**0011,01**	5	3-2	202	39	494	157	20	20	8	8
KBPKN	**0013.10**	5	3-2	20	5	23	86	30	30	3	4
KNNKP	**0002.01**	5	3-2	60,080	12,023	1	1	50	50	11	11
KNNKQ	**3002.00**	5	3-2	2,387	465	6,352	2,010	1	0	50	50
KNPKN	**0004.10**	5	3-2	2	25	27	132	30	29	3	4
KNPKQ	**3001.10**	5	3-2	2,459	4	1	1	3	3	43	43
KPPKP	**0000.21**	5	3-2	1	3	1	4	21	21	7	7
KPPKQ	**3000.20**	5	3-2	8	15	19	16	6	5	29	29
KQPKP	**1000.11**	5	3-2	69	2	1,024	7,412,631	5	5	1	1
KQPKQ	**4000.10**	5	3-2	1,595	2,415	2	4	50	50	15	16
KQRKP	**1100.01**	5	3-2	76,181	2,592	683	892,287	3	3	1	1
KQRKQ	**4100.00**	5	3-2	23	156	1	2	50	50	8	9
KRBKR	**0410.00**	5	3-2	1,041	175	3	14	50	50	3	4
KRPKB	**0130.10**	5	3-2	130	254	502	1,672	50	50	1	2
KRPKP	**0100.11**	5	3-2	20	22	17	18	9	9	10	10
KRPKQ	**3100.10**	5	3-2	2	5	9,275	4,898	9	8	50	50

Table 3. Chess Endgames: 3-to-5-man data where $EZ_{50} \neq EZ$.

Endgame				DTZ Metric: # of maximal positions				DTZ Metric: max depth, moves			
				1-0		0-1		1-0		0-1	
	GBR	**#**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBBKNN	**0026.00**	6	3-3	11	1	488	1,518	38	38	3	4
KQQKBB	**2060.00**	6	3-3	984	5,128	137	714	6	6	3	4
KQQKNN	**2006.00**	6	3-3	2	8	1	36,110	7	7	1	1
KQQKQR	**5300.00**	6	4-2	4	2	1	12	48	47	56	56
KRRKRB	**0530.00**	6	3-3	22	13	1	455	54	54	6	6
KBBBKN	**0093.00/30**	6	4-2	6	6	951	4,838	12	12	0	1
KBBBKQ	**3090.00/30**	6	4-2	1	9	1	3	10	9	51	51
KBBNKN	**0024.00**	6	4-2	9	54	3,663	18,984	31	31	0	1
KBNNKN	**0015.00**	6	4-2	17	56	4,335	22,890	28	28	0	1
KBNNKQ	**3012.00**	6	4-2	5	1	1	4	12	11	49	49
KNNNKQ	**3009.00/30**	6	4-2	1	1	6	11	9	8	35	35
KQNNKQ	**4002.00**	6	4-2	2	2	5	20	71	71	13	14
KRNNKQ	**3102.00**	6	4-2	2	1	2	3	28	27	41	41

Table 4. Chess Endgames: some 6-man DTZ data.

Endgame				DTZ_{50} Metric: # of maximal positions				DTZ_{50} Metric: max depth, moves			
				1-0		0-1		1-0		0-1	
	GBR	**#**	**w-b**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**	**wtm**	**btm**
KBBKNN	**0026.00**	6	3-3	46	17	488	1,518	29	28	3	4
KQQKBB	**2060.00**	6	3-3	1	5	137	714	8	8	3	4
KQQKNN	**2006.00**	6	3-3	2	8	1	36,110	7	7	1	1
KQQKQR	**5300.00**	6	4-2	4	2	6	26	48	47	50	50
KRRKRB	**0530.00**	6	3-3	372	107	1	455	50	50	6	6
KBBBKN	**0093.00/30**	6	4-2	3	6	951	4,838	14	14	0	1
KBBBKQ	**3090.00/30**	6	4-2	1	9	11	15	10	9	50	50
KBBNKN	**0024.00**	6	4-2	9	54	3,663	18,984	31	31	0	1
KBNNKN	**0015.00**	6	4-2	3	3	4,335	22,890	29	29	0	1
KBNNKQ	**3012.00**	6	4-2	5	1	1	4	12	11	49	49
KNNNKQ	**3009.00/30**	6	4-2	1	1	6	11	9	8	35	35
KQNNKQ	**4002.00**	6	4-2	10,534	9,796	5	20	50	50	13	14
KRNNKQ	**3102.00**	6	4-2	2	1	2	3	28	27	41	41

Table 5. Chess Endgames: some 6-man DTZ_{50} data.

The following lines, starting from some positions listed in Table 7 below, show strategies variously retaining the win, failing to retain the win, repeating positions to draw or being suboptimal. They include an established notation showing the criticality of the moves:

" ≡ *unique value-preserving move*; ' ≡ *only optimal move*; º ≡ *only legal move*.

KBBKP position BB-P1 – *dtz* = 1m; dtz_{50} = 7m:

Sφ ¯ Sσ, σ = C^-, M^- or Z^-: 1. ... a1Q+?? {*dtz* = 51m; White can force a 50m draw} ½-½.

SZ_{50}^+ ¯ SZ_{50}^-: 1. ... Kc4" 2. Bf3+ Kc3 3. Be1+' Kd4" 4. Bf2+' Ke5' 5. Bg3+' Kf6' 6. Bh4+' Kg7' {*dtm* = 17m} 0-1.

KNNKP position NN-P1 – *dtz* = 20m, *dtc* = 63m, *dtm* = 64m, dtz_{50} = 44m:

$S(C, M, Z)^-\sigma$ ¯ SZ_{50}^+: 1. Ng1?? h3" {*dtz* = 61m; Black can force a 50m draw} ½-½.

SZ_{50}^- ¯ SZ_{50}^+: 1. Ngf2' Ke3' 2. Kc3' Ke2' 3. Kd4' Kd2' 4. Ne4+' Ke2' 5. Neg5' Kd2' 6. Nf3+' Ke2' 7. Ke4' Kf1' 8. Kd3 Kg2º 9. Nfg5' Kg3' 10. Ke3 Kg4' 11. Ke4' Kg3' ... 1-0.

KNNKP position NN-P2 – *dtz* = 1m, dtz_{50} = 43m:

SZ^-σ ¯ Sτ: 1. Nbc4'?? {*dtz* = 58m; Black can force a 50m draw} ½-½.

S(C/M)$^-$σ ¯ SZ_{50}^+: 1. Na4" {dtz_{50} = 42m, *dtm* = 88m} Kd2º 1-0.

SZ_{50}^- ¯ SZ_{50}^+: 1. Na4" Kd2° 2. Nc4+ Kd3' 3. Ncb2+ Kd2 4. Kb1 Ke3' 5. Kc1 Ke2 6. Kc2' Ke3' 7. Kc3' Ke4' 8. Nd3 Ke3 9. Ndc5' Kf4' 10. Kd4' Kf5' 11. Nd3 Ke6 12. Ke4' Kd6 13. Nf4 Kc6 14. Ne2' Kd6' 15. Nd4' Ke7' 16. Ke5' Kf7' 17. Kf5' Ke7 18. Nb5' Kd7 19. Ke5' Kc6' 20. Na3' Kd7' 21. Nc5+' Ke7' 22. Ne4' Kf7' 23. Kd6' Kg7 24. Ke6' Kg6' 25. Nc4' Kg7' 26. Ned2 Kg6 27. Nf3' Kh6 28. Kf5' Kg7 29. Ng5' Kf8 30. Kf6' Ke8' 31. Ke6' Kf8' 32. Nh3' Kg8' 33. Nf4 Kg7' 34. Ke7' Kh6 35. Kf6' Kh7' 36. Ne2 Kh6 37. Ng3' Kh7' 38. Nf5' Kg8 39. Ke7' Kh8' 40. Ne5 Kh7 41. Ke8 Kg8 42. Ng6' Kh7' 43. Kf7' a4º {*dtm* = 3m} 1-0.

KNNKP position NN-P3 – *dtz* = 1m, dtz_{50} indicates 'draw', *dtr* = 51m, dtz_{51} = 31m:

SZºσ - Sφ, σ = C$^-$, M$^-$, Z$^-$ or Z_{50}^-: 1. Kc2? {*dtr* > 51m}.

SZºH_{50}^- - SH_{50}^+: 1. Nb1+' {*dtr* = 51m, controlling DTR} Ka4'

KNNKP position NN-P4 – *dtz* = 16m, dtz_{50} indicates 'draw', dtz_{51} = 25m, *mleft* = 25m:

SZºH_{50}^- – SH_{50}^+: 1. Nd5+? {dtz_{51} = 26m} Kc4' 2. Ndc3 Kb4' {NN-P4 repeated} ½-½.

KQPKQ position QP-Q1 - *dtc* = 52m, *dtz* = 1m, dtz_{50} = 50m:

Sσ ¯ Sτ, σ = C$^-$, M$^-$ or Z$^-$: 1. b7'?? {*dtz* = 51m; Black can force a 50m draw} ½-½.

SZ_{50}^- - SZ_{50}^+: 1. Qg5" Qe4' 2. Kc5" Qc2+' 3. Kd5 Qb3+ 4. Kc6' Qe6+' 5. Kc5' Qc8+' 6. Kd4' Qh8+' 7. Kc4' Qh7 8. Qd5' Qc2+' 9. Kb4 Qb2+' 10. Kc5' Qa3+ 11. Kc6' Qa4+' 12. Kd6 Qf4+' 13. Kd7' Qg4+' 14. Qe6' Qg7+' 15. Kd6' Qg3+' 16. Kc5' Qg5+' 17. Kc4 Qc1+' 18. Kd5' Qb2' 19. Qg6' Qb5+' 20. Kd4" Qb4+' 21. Ke5' Qc5+' 22. Kf4" Qd4+' 23. Kf5 Qc5+' 24. Kg4' Qd4+' 25. Kh5' Qd5+' 26. Kh6' Ke1' 27. Qg1+' Ke2' 28. Qg4+' Kf1' 29. Qg5' Qc6+ 30. Qg6" Qb7' 31. Qf6+' Ke2' 32. Kg5' Ke3' 33. Qe5+ Kf2' 34. Qc5+' Ke2' 35. Kf4' Qf3+' 36. Ke5' Qg3+ 37. Ke6' Qh3+ 38. Kd6' Qh6+' 39. Kc7' Qg7+ 40. Kc6' Qf6+ 41. Qd6' Qc3+' 42. Kd7' Qf3' 43. Kc8 Qc3+ 44. Kd8' Qa5' 45. Ke7' Qb5 46. Qf6' Qb1 47. Kf7 Qh7+ 48. Kf8' Qb1 49. Qe7+' Kd1 50. b7' {*dtm* = 21m} 1-0.

KRPKP position RP-P2 – *dtz* = 1m, dtz_{50} = 6m:

Sφ ¯ Sστ, σ = C$^-$, M$^-$ or Z$^-$: **1. ... g1Q'??** {*dtr* > 50m; White can force a 50m draw} ½-½.

SZ_{50}^+ ¯ SZ_{50}^-: 1. ... Kb2" 2. Rb4+' Kc2" 3. Rc4+' Kd2' 4. Rd4+' Ke2' 5. Re4+' Kf2" 6. Re7 g1Q" {*dtm* = 49m} 0-1.

KRPKQ position RP-Q1 – *dtz* = 2m, dtz_{50} = 21m:

Sφ ¯ Sστ, σ = C$^-$ or M$^-$: **1. ... Qd6+'??** {*dtr* > 50m; White can force a 50m draw} ½-½.

Sφ ¯ SZ^-τ: **1. ... Qe4+'??** {*dtr* > 50m; White can force a 50m draw} ½-½.

SZ_{50}^+ ¯ SZ_{50}^-: 1. ... Qe6+" 2. Kg5' Qg8+" 3. Kh6' Qd5' 4. Rg7' Qh1+" 5. Kg6' Qg1+' 6. Kf7' Qf1 7. Rg6+' Kb7' 8. Rf6' Qg2 9. Ke6 Qe4+' 10. Kd6 Kb6 11. Rf7' Kb5' 12. Rf6 Kc4' 13. Rf7' Kd4' 14. Rf8' Qd5+' 15. Ke7' Qc5+' 16. Kf7 Kd5" 17. Kg7 Qg1+' 18. Kf6 Qg4' 19, Ke7' Qe6+' 20. Kd8º Kc6 21. Rf6 Qxf6+" {*dtm* = 2m} 0-1.

KBBKNN position BB-NN1 - *dtz* = 1m, dtz_{50} = 28m:

Sστ ¯ Sφ, σ = C$^-$, M$^-$ or Z$^-$: **1. Bxg6'??** {*dtz* = 54m; Black can force a 50m draw} ½-½.

SZ_{50}^- ¯ SZ_{50}^+: 1. Bd6" Nh8' 2. Bc6+" Ka5° 3. Kb3" Nc1+' 4. Kc4" Nf7' 5. Bc7+" Ka6° 6. Bd5" Nh8' 7. Bf3' Ng6' 8. Bd6" Nh4' 9. Be4" Ne2' 10. Bh2" Ka5' 11. Bc7+' Ka6' 12. Kc5' Ka7' 13. Bd3' Ng1' 14. Bg3 Ng2' 15. Kc6' Nh3' 16. Bf1' Nhf4' 17. Bf2+" Kb8' 18. Bb6' Ka8' 19. Ba6' Kb8' 20. Bc4' Nh5' 21. Bc7+' Ka7 22. Be5' Nhf4' 23. Bd6' Nh5 24. Kc7' Nf6' 25. Bc5+' Ka8° 26. Bb5 Nd5+' 27. Kc8" Ne1 28. Bc6#' 1-0.

KQQKBB position QQ-BB1 - *dtz* = 2m, dtz_{50} = 7m:

SZ^- ¯ Sφ: 1. Qxd4+'?? Bxd4+ {*dtz* = 67m; Black can force a 50m draw} ½-½.

SZ_{50}^- - SZ_{50}^+: 1. Kb1" Be4+' 2. Ka2" Bd5+' 3. Ka3' Bd6+' 4. Ka4' Bc6+' 5. Ka5 Kc3 6. Qc1+' Kb3 7. Qxc6' {*dtm* = 2m} 1-0.

KBNNKQ position BNN-Q1 - *dtz* = 1m, dtz_{50} = 36m:

$S\varphi$ ¯ $S\sigma$, σ = C^-, M^- or Z^-: 1. ... Qxa1'?? {*dtz* = 52m; Black can force a 50m draw} ½-½.

SZ_{50}^- ¯ SZ_{50}^+: 1. ... Qh7+" 2. Kd2' Qd7+" 3. Kc3' Ke2' 4. Bb2' Qg4" 5. Kb3' Qe6" 6. Kc3' Qe4" 7. Kb3' Qg4' 8. Kc3' Qf4' 9. Kb3' Qb8+' 10. Kc2' Qb4' 11. Na3' Qe4+" 12. Kb3' Qd5+' 13. Kc3' Qf3+' 14. Kc4' Kd1 15. Kb4' Qb7+" 16. Nb5' Kc2' 17. Bd4' Qe7+' 18. Kc4' Qe6+' 19. Kc5' Qf5+' 20. Kc4' Qc8+' 21. Kb4' Qf8+' 22. Ka4 Qg8' 23. Kb4 Kd3' 24. Bc3' Qd5' 25. Bd4 Qc4+' 26. Ka5' Qg8' 27. Ka4' Qa8+' 28. Kb4' Qf8+' 29. Kb3' Qe7' 30. Bb2' Qe6+' 31. Ka4 Qa2+ 32. Ba3' Qc4+' 33. Ka5 Qd5' 34. Kb4' Qe4+ 35. Ka5 Qa8+' 36. Kb6 Qxh8 {*dtm* = 22m} 0-1.

KQNNKQ position QNN-Q1 - *dtz* = 3m, dtz_{50} = 4m, *dtm* = 5m:

SZ^- ¯ SZ^+: **1. Qa3+'??** Kd1' 2. Qa1+" Ke2º 3. Qxh1" {*dtz* = 52m} ½-½.

SZ_{50}^- ¯ SZ_{50}^+: 1. Qe3+" Kb1' 2. Qb6+" Kc1' 3. Qb2+' Kd1º 4. Qd2# 1-0.

Endgame	res.	# extra draws wtm	# extra draws btm	# delayed wtm	# delayed btm	% of nominal wins: extra draws wtm	extra draws btm	delayed wtm	delayed btm
KBBKN	1-0	3,993,656	7,852,543	0	0	21.05	48.20	0	0
KBBKP	1-0	171	687	3,889	1,800	ε	ε	0.01	ε
	0-1	119,226	1,444,441	1,524	3,741	5.85	8.47	0.07	0.02
KBBKQ	0-1	2,154,114	490,797	0	0	8.49	1.46	0	0
KBNKN	1-0	139,893	72,483	0	0	0.52	1.93	0	0
KBNKP	1-0	185	275	1,641	1,685	ε	ε	ε	ε
KBPKN	1-0	257	264	602	1,530	ε	ε	ε	ε
KNNKP	1-0	10,684,968	9,495,721	17,093,973	6,239,778	26.35	46.87	42.16	30.80
	0-1	4,255	10,877	301	357	0.14	0.06	0.01	ε
KNNKQ	0-1	11,990	3,667	0	0	0.05	0.01	0	0
KNPKN	1-0	61	86	48	39	ε	ε	ε	ε
KNPKQ	0-1	1	0	0	0	ε	0	0	0
KPPKP	1-0	1,834	2,062	149	55	ε	ε	ε	ε
KPPKQ	1-0	1,641	3	0	0	0.01	0.01	0	0
KQPKP	1-0	19	3,266	2,664	2,207	ε	ε	ε	ε
KQPKQ	1-0	28,468	22,411	42,756	28,526	0.02	0.08	0.03	0.10
KQRKP	1-0	0	79	0	0	0	ε	0	0
KQRKQ	1-0	230	1,106	0	0	ε	ε	0	0
KRBKR	1-0	2,263	725	0	0	0.01	0.02	0	0
KRPKB	1-0	35	83	53	74	ε	ε	ε	ε
KRPKP	1-0	0	240	124	33	0	ε	ε	ε
	0-1	679	12,137	26	30	0.14	0.05	0.01	ε
KRPKQ	1-0	1,592	1	116	0	ε	ε	ε	0
	0-1	72,802	29,723	26,336	9,097	0.06	0.02	0.02	ε
KBBKNN	1-0	141,874,223	38,562,549	4,961,624	1,402,773	50.15	70.98	1.75	2.58
KQQKBB	1-0	23,343	6,776,509	1,244,572	5,432,160	ε	0.58	0.18	0.47
KQQKNN	1-0	130	44,687	4,704	22,000	ε	ε	ε	ε
KQQKQR	0-1	17,313	41,775	42,552	66,504	0.02	0.01	0.04	0.01
KRRKRB	1-0	380	145	0	0	ε	ε	0	0
	0-1	396	11,281	30	799	0.02	0.03	ε	ε
KBBBKN	1-0	743,762	37,035,833	55,589,963	161,070,140	0.15	6.16	11.28	26.80
KBBBKQ	0-1	21,650,797	31,223,711	6,004,068	11,096,464	15.04	6.15	4.17	2.19
KBBNKN	1-0	640,358	36,582,112	136,891,517	318,970,567	0.03	1.74	6.44	15.17
KBNNKN	1-0	96,123	1,016,653	10,322,215	13,062,956	ε	0.05	0.46	0.70
KBNNKQ	0-1	178,774	178,631	179,015	143,015	0.03	0.01	0.03	0.01
KNNNKQ	0-1	125,488	181,848	91,063	99,907	0.09	0.04	0.07	0.02
KQNNKQ	1-0	49,329	38,050	0	0	ε	0.01	0	0
	0-1	1,538	206,733	0	2	0.04	0.05	0	ε
KRNNKQ	0-1	33,448	252,183	10,270	30,764	0.04	0.03	0.01	ε

Table 6. The impact of the 50-move drawing rule.

Key		Position	stm	depth in plies					Notes
				dtc	dtm	dtr	dtz	dtz_{50}	
$EZ_{50} \neq EZ$									
BB-N	1-0	8/8/8/7B/4k3/4B3/3K4/1n6	w	119	143	119	119	—	q.v. BB-P
BB-P	1-0	8/8/8/7B/4k3/4B3/1p1K4/8	b	6	144	119	6	—	1. ... b1=N+ {BB-N}
	0-1	8/8/6B1/3K4/5B2/8/p7/3k4	b	1	157	136	1	—	1. ... a1=Q" {BB-Q}
BB-Q	0-1	8/8/6B1/3K4/5B2/8/8/q2k4	w	136	156	136	136	—	q.v. BB-P
BN-N	1-0	8/8/3K4/8/8/3B4/k7/1n1N4	w	139	199	139	139	—	q.v. BN-P
BN-P	1-0	8/8/3K4/8/8/3B4/kp6/3N4	b	9	200	139	9	—	1. ... b1=N {BN-N}
BP-N	1-0	1n6/3P4/8/8/1K6/7B/8/k7	w	1	199	138	1	—	1. d8=N" {*dtz*=138p}
NN-P	1-0	K1k5/3N1N2/8/8/4p3/8/8/8	w	169	169	164	164	—	maxDTZ pos.
	0-1	3k3N/3N4/3K4/8/8/8/7p/8	b	1	145	126	1	—	1. ... h1=Q" {NN-Q}
NN-Q	0-1	3k3N/3N4/3K4/8/8/8/8/7q	w	126	144	126	126	—	q.v. NN-P
NP-N	1-0	kn6/3P4/1K6/8/8/8/3N4/8	w	1	191	130	1	—	1. d8=B" {*dtz*=130p}
NP-Q	0-1	1k1K4/4P1N1/8/8/8/6q1/8/8	w	6	124	103	6	—	1. e8=N {*dtz*=103p}
PP-P	1-0	8/4P3/8/8/8/4P3/kp1K4/8	b	2	244	102	2	—	1. ... b1=Q {PP-Q}
PP-Q	1-0	8/4P3/8/8/8/4P3/k2K4/1q6	w	1	243	102	1	—	1. e8=Q" {QP-Q}
QP-P	1-0	8/4Q3/8/8/8/K7/6Pp/5k2	w	5	191	?	1	—	
QP-Q	1-0	4Q3/8/8/8/8/4P3/k2K4/1q6	b	222	242	102	102	—	q.v. PP-Q
QR-P	1-0	Q7/2k5/8/8/8/8/R2p4/K7	b	2	134	119	2	—	1. ... d1=Q {QR-Q}
QR-Q	1-0	Q7/2k5/8/8/8/8/R7/K2q4	w	119	133	119	119	—	q.v. QR-P
RB-R	1-0	8/3B4/8/1R6/5r2/8/3K4/5k2	w	117	129	117	117	—	maxDTZ pos.
RP-B	1-0	K1R5/8/3k4/3P4/8/8/1b6/8	w	113	131	105	105	—	maxDTZ pos.
RP-P	1-0	6R1/P6K/1k6/8/8/8/3p4/8	b	1	136	120	1	—	1. ... d1=Q" {*dtz*=120p}
	0-1	8/8/8/5PR1/8/2K5/5p2/k7	w	2	188	130	2	—	1. Kd4" f1Q" {*dtz*=130p}
RP-Q	1-0	6R1/P7/2q5/2k5/8/8/8/6K1	b	2	118	102	2	—	only frustrated btm 1-0 pos.
	0-1	8/7R/6K1/8/5P2/8/8/k6q	b	116	165	107	3	—	
BB-NN	1-0	8/6B1/8/8/2B1n3/6K1/3k3n/8	w	1	147	122	1	—	1. Kxh2" {*dtz*=122p}
QQ-BB	1-0	8/8/8/4b3/8/Q7/2k1b3/K5Q1	w	2	143	121	2	—	1. Qc3" Bxc3+ {*dtz*=121p}
QQ-NN	1-0	8/8/8/8/1Q6/3n4/2n3k1/K3Q3	w	3	135	113	3	—	1. Kb1" Ncxe1 {*dtz*=113p}
QQ-QR	0-1	8/Q7/1Q6/8/r7/8/8/qK5k	w	2	132	116	2	—	1. Kc2º Rxa7" {*dtz*=116p}
RR-RB	1-0	3R4/8/R7/8/8/8/6r1/k3K2b	b	102	122	102	102	—	
	0-1	8/R7/8/4b3/8/1r6/R7/K3k3	w	2	116	102	2	—	1. Rb2º Rxb2" {*dtz*=102p}
BBB-N	1-0	8/8/8/8/8/8/2B1n3/K1k3BB	w	2	145	119	2	—	1. Bb6 Kxc2 {*dtz*=119p}
BBB-Q	0-1	8/8/8/8/q7/3BB3/8/K2kB3	w	2	142	120	2	—	1. Kb2 Kxe1 {*dtz*=120p}
BBN-N	1-0	8/8/8/8/8/8/2N2B2/K1kn3B	w	2	141	115	2	—	1. Bb6 Kxc2 {dtz=115p}
BNN-N	1-0	n7/8/8/8/8/6B1/6N1/K4kN1	w	2	181	119	2	—	1. Ne3+" Kxg1º {*dtz*=119p}
BNN-Q	0-1	q7/8/8/8/8/N7/3N4/K1kB4	w	2	126	104	2	—	1. Ndc4 Kxd1" {*dtz* = 104p}
NNN-Q	0-1	8/8/2q5/8/8/N7/3N4/K1kN4	w	2	126	104	2	—	1. Ndc4 Kxd1" {dtz = 104p}
QNN-Q	1-0	7q/1Q6/8/5N2/8/8/8/K1k4N	w	101	107	101	101	—	1. Ng7" ...
	0-1	8/8/8/1N6/8/8/N7/kqK2Q2	w	2	124	104	2	—	1. Kd2º Qxf1" {*dtz*=104p}
RNN-Q	0-1	8/8/1R6/q7/3N4/8/4N3/K2k4	w	2	122	102	2	—	1. Kb2 Qxb6+" {*dtz* = 102p
Strategy Failure Positions									SZ_{50} ok if dtz_{50} cited
BB-P1	1-0	8/8/8/1k6/8/8/p4BB1/3K4	b	1	123	58?	1	13	S(Cσ/M/Zσ) ×
NN-P1	1-0	8/8/8/8/2K3Np/7N/3k4/8	w	126	127	40	88	88	S(C/M/Z)σ ×
NN-P2	1-0	8/8/1N6/p7/8/4N3/8/K1k5	w	176	177	100?	2	86	SZσ ×; S(C/M) ok
NN-P3	1-0	8/8/8/2pN4/8/k1N5/8/2K5	w	115	115	102	2	—	$SZ^{o}H_{50}^{-}$: 1. Nb1+'
NN-P4	1-0	8/8/8/2p5/1k6/2N5/2K5/1N6	w	113	113	102	32	—	$SZ^{o}H_{50}^{-}$ repeats positions
QP-Q1	1-0	8/8/1P5Q/1K6/3q4/8/5k2/8	w	103	125	99	1	99	S(C/M/Z)σ ×
RP-P1	1-0	6R1/8/Pk6/8/8/8/p2K4/8	w	3	31	26?	1	5	S(C/Z)σ ×; SM ok
RP-P2	1-0	8/8/5K2/8/2R2P2/8/6p1/k7	b	1	159	?	1	11	S(C/M/Z)σ ×
RP-Q1	1-0	8/4q2R/k5K1/8/5P2/8/8/8	b	113	163	?	3	41	S(C/M/Z)σ ×
BB-NN1	1-0	8/8/6n1/8/k3BB2/8/n1K5/8	w	1	133	55	1	55	S(C/M/Z)σ ×
QQ-BB1	1-0	8/Q7/8/3bb3/8/8/3k4/K4Q2	w	3	17	13	3	13	SZ^{-} ×
BNN-Q1	0-1	7N/6q1/8/8/2N5/3K1k2/8/B7	b	1	125	?	1	71	S(C/M/Z)σ ×
QNN-Q1	0-1	8/2N5/8/2q5/5N2/2k5/8/2K4Q	b	5	9	7	5	7	S(C/Z)σ ×; SM ok

Table 7. Example Positions.[7]

[7] Without a DTR EGT, it is not always possible to determine *dtr* precisely.