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# Chess endgames: 6-man data and strategy

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## Abstract

While Nalimov's endgame tables for Western Chess are the most used today, their Depth-to-Mate metric is not the most efficient or effective in use. The authors have developed and used new programs to create tables to alternative metrics and recommend better strategies for endgame play.

*Key words:* chess: conversion, data, depth, endgame, goal, move count, statistics, strategy

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## 1 Introduction

Chess endgames tables (EGTs) to the 'DTM' Depth to Mate metric are the most commonly used, thanks to codes and production work by Nalimov [10,7]. DTM data is of interest in itself, even if *conversion*, i.e., change of force, is more often adopted as an interim objective in human play. However, more effective endgame strategies using different metrics can be adopted, particularly by computers [3,4]. A further practical disadvantage of the DTM metric is that, as maxDTM increases, the EGTs take longer to generate and are less compressible.

Here, we focus on metrics DTC, DTZ<sup>1</sup> and DTZ<sub>50</sub><sup>2</sup>; the first two were effectively used by Thompson [19], Stiller [14], and Wirth [20]. New programs by Tamplin [15] and Bourzutschky [2] have already enabled a complete suite of 3-to-5-man DTC/Z/Z<sub>50</sub> EGTs to be produced [18]. This note is an update, focusing solely on Tamplin's continuing work, assisted by Bourzutschky, with the latter code on 6-man, pawnless endgames for which DTC  $\equiv$  DTZ and DTC<sub>50</sub>  $\equiv$  DTZ<sub>50</sub>. Section 2 outlines the algorithm used. Sections 3 and 4 review the new DTZ and DTZ<sub>50</sub> data tabled in the Appendix. In section 5, *endgame strategy* is defined and improved strategies are recommended for the 50-move and *k*-move contexts.

## 2 The NBT code

Here, we review the algorithm and the 'NBT' code developed in turn by Nalimov, Bourzutschky and Tamplin. The first author extended Nalimov's DTM-code to enable it to generate EGTs to metrics DTC<sub>(k)</sub>, DTM<sub>k</sub> and DTZ<sub>(k)</sub><sup>3</sup>. This involved generalising some DTM-specific aspects of the algorithm, as well as making the obvious changes to the iterative formula for deriving depth. For DTC<sub>(k)</sub>, the code retains the efficiencies of the DTM-code while requiring maxDTC rather than maxDTM cycles<sup>4</sup>. Because EGT

<sup>1</sup> DTC  $\equiv$  *Depth to Conversion*, i.e., to force change and/or mate. DTZ  $\equiv$  *Depth to (Move-Count) Zeroing (Move)*, i.e., to Pawn-push, force change and/or mate – when a move-counter is set to zero again.

<sup>2</sup>  $dtz_k = dtz$  unless a *k*-move rule allowing a draw-claim sets a value of *draw*.

<sup>3</sup> The board-size, piece-type and rule generalizations also effected are not covered here.

<sup>4</sup> An advantage, as, e.g., KQBNKN has maxDTM = 107 but maxDTC = 6.

generation to the DTZ metric has not been implemented generically as a sequence of ‘fixed pawn structure’ sub-EGT generations, this is not so for  $DTZ_{(k)}$  computations. The second author ran the code on single- and multi-processor UNIX systems, and evolved the code to:

- increase portability as Nalimov’s C++ is non-standard and Windows-oriented,
- manage virtual stores and files greater than 2GBytes,
- accumulate integer counts greater than  $2^{31}-1$ ,
- pursue EGT depths  $> 126$ , requiring 16-bit database entries, and
- synchronise multiple processes more rigorously.

Experience confirms the observation [13] that manual file-management can be a source of error. This suggests that the Nalimov file-format should include a file-header to help prevent such errors with details, e.g., of author, code version, metric, degree and date of completion, and compression algorithm.

Table 1. Examples of extreme, atypical maxDTC wins and losses.

| Endgame | Result | Position                     | maxDTC | avgeDTC | maxDTC/avgeDTC |
|---------|--------|------------------------------|--------|---------|----------------|
| KRPKN   | 0-1    | K1k5/8/Pn6/8/R7/8/8/8 w      | 1      | 0.01    | 98.00          |
| KRBNKQ  | 1-0    | 1k4q1/8/N2K4/8/8/8/R3B3 b    | 98     | 1.31    | 74.96          |
| KRBNKQ  | 1-0    | 1k4q1/8/3K4/8/1N6/8/8/R3B3 w | 99     | 1.33    | 74.45          |
| KQRKQR  | 1-0    | 4q3/7R/7Q/4r3/4k3/8/8/2K5 w  | 92     | 1.92    | 48.03          |
| KQPKN   | 0-1    | K1k5/8/Pn6/8/Q7/8/8/8 w      | 1      | 0.02    | 47.00          |
| KRBKR   | 1-0    | 8/3B4/8/1R6/5r2/8/3K4/5k2 w  | 59     | 1.4     | 42.13          |

Table 2. Chess EGTs: comparative file sizes.

|         |          | DTM<br>MB | DTC<br>% | DTZ<br>% | DTZ <sub>50</sub><br>% | $\delta(DTZ_{50}, DTZ)$<br>% | DTZ + ‘ $\delta$ ’<br>% |
|---------|----------|-----------|----------|----------|------------------------|------------------------------|-------------------------|
| 3-5-man | pawnless | 1,822     | 71.29    | 71.29    | 0.00                   | 0.00                         | 71.29                   |
| 3-5-man | with Ps  | 5,579     | 59.14    | 43.36    | 15.36                  | 0.70                         | 44.06                   |
| 3-5-man | all      | 7,401     | 62.13    | 50.24    | 11.58                  | 0.53                         | 50.77                   |
| 3-3/4-2 | pawnless | 220,623   | 56.37    | 56.37    | 20.56                  | 1.12                         | 57.50                   |
| 3-6-man | to date  | 228,024   | 56.56    | 56.17    | 20.27                  | 1.11                         | 57.28                   |

### 3 The DTC and DTZ metrics

DTC EGTs are interesting, not only for completeness, but because *conversion* is an intuitively obvious objective and the DTC EGTs document precisely the phase of play when the material nominated is on the board. The DTZ metric is more important than DTC, being necessary if the length of the current phase of play is to be *guarded* in the context of chess’  $k$ -move rule,  $k$  currently being 50. Where no Pawns are involved, as here,  $DTZ \equiv DTC$ .

The NBT-code measures depth consistently in *winner’s moves* and does not assume that conversion is effected by the winner. Also, it does not allow the loser to make a voluntary, ‘natural’ if unavailing capture, e.g., {wKe1Qf1Rb1/bKa1 b: 1. ... Kxb1}. The ICGA website (2004) provides the latest data, including %-wins and average win-length. Because there are many wins in 1, the % of positions won does not characterise well the presence of wins in an endgame. Similarly, maxDTx is not a good indicator of typical DTx and Table 1 gives some maxDTC positions for endgames with extreme maxDTC/averageDTC. We therefore calculate a new characteristic,

$$x\text{-Presence} \equiv \% \text{ of\_decisive\_positions} \times (\text{Average DTx})$$

$x$ -Presence may be compared with  $\max DTx$  and %-wins [8]. It is not unduly affected by the wins in 1 or by the long tail of deep wins, and is the number of moves for which a win is expected to be on the board when  $DTx \equiv DTC$ .

### 3.1 A Review of the DTZ data

The results are in the Appendix, Table 3. These agree with the earlier results of Stiller [14] and Thompson [17] with two exceptions.<sup>5, 6</sup> Note that legal but unreachable positions can affect the statistics.<sup>7</sup>

KBNK wtm wins had the largest C-presence (2455.76) of 3-5-man endgames with density 99.51% and average DTC 24.68. Only KRBKNN btm losses exceed this (4068.54) with density 57.52% and average DTC 70.73.

Table 2 summarises the absolute and comparative sizes of the various EGTs.

## 4 The $DTZ_{50}$ metric

The  $DTZ_{50}$  metric rates as wins only those positions winnable against best play given the 50-move rule. Figure 1 shows those 5-man endgames for which some DTZ and  $DTZ_{50}$  depths differ<sup>8</sup>, thereby affecting the value or depth of some 6-man positions. Let  $KwKb$ , e.g., KBBKN, be an endgame with wtm and btm 1-0 wins impacted by the 50-move rule. Then the  $DTZ_{50}$  EGTs for  $KwxKb$  and  $KwKby$ , e.g., KQBBKN and KBBKNN, must be computed and are likely to differ from their DTZ equivalents.

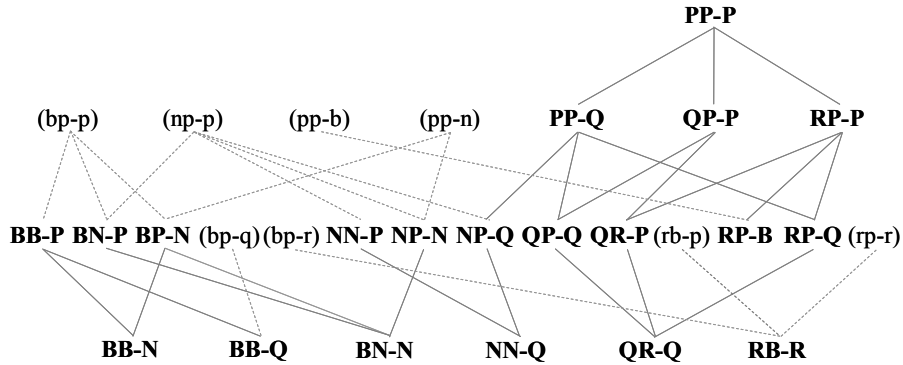


Figure 1. 5-man endgames with  $EZ_{50} \neq EZ$ .

Table 4 in the Appendix lists 6-man  $DTZ_{50}$  EGT data for endgames where  $EZ_{50} \neq EZ$ . Table 5 summarises 50-move impact, minimal for KRRKRB (1-0), considerable for KBBKNN. Table 6 gives an example position for each affected endgame. 63 of the 135 6-man pawnless endgames are affected by the 50-move rule. Although  $DTZ_{50} \geq DTZ$ ,  $\max DTZ_{50}$  is rarely greater than  $\max DTZ$ : KQNKBB, KQQKBB, KBBBKN and KBNNKN are the only examples to date. Wins frustrated by the 50-move rule produce a

<sup>5</sup> Their  $\max DTC$  for KQNKRR and KQNNKQ is 1 greater: in both cases, Black is forced to convert.

<sup>6</sup> For KBNNKN [17], '27' should be '28': a foreshortened line went unseen.

<sup>7</sup> e.g., KQQKNN has '1 wtm loss in 1' in 8/8/8/8/1n6/QQn5/K2k4 w. The double-check is impossible.

<sup>8</sup> Endgames where DTZ and  $DTZ_{50}$  might have differed, but did not, are bracketed in lower-case.

$\max DTZ_{50} < \max DTZ \leq 50$  for only KBBKBN and KBBKNN so far. KBBKNN has the majority of its wins frustrated, and relatively few wins can be retained by a deeper strategy in the current phase. Here, the 50-move rule bars the now well defined route to many KBBKN wins [12]. There are significant percentages of frustrated wins in KBBxKQ (0-1), and of delayed 1-0 wins in KBBxKN. Elsewhere, the 50-move impact is sparsely distributed and one might expect that this becomes sparser as the number of men increases.

Note that, as  $DTZ_{50} \geq DTZ$  for a decisive position, we may construct an EGT coding,  $EdZ_{50}Z$ , of  $\delta(DTZ_{50}, DTZ)$ <sup>9</sup> enabling  $DTZ_{50}$  to be derived from  $DTZ$  and  $EdZ_{50}Z$ . The latter notes only  $DTZ_{50}-DTZ$  for the delayed wins, and ‘new draws’ when  $DTZ \leq 50$ :  $DTZ > 50$  already implies ‘new draw’. If  $EdZ_{50}Z$  is *null*, it is not required. For 3-5-men, these EGTs are only 0.53% the size of the corresponding DTM EGTs. They can in fact be made much smaller by designer-compression techniques more tailored to the data than the established compression method adopted by Nalimov.

## 5 Endgame strategies

An *endgame strategy*, denoted here by  $S_s$ , is an algorithm for filtering the available moves to a preferred choice. Endgame strategies can be applied in sequence.  $S_{s_1}s_2 \dots s_n$  denotes a compound endgame strategy using strategies  $S_{s_1}, S_{s_2}, \dots, S_{s_n}$  in turn. Let  $dtx$  be the depth by metric  $DTx$ , and  $Ex$  an EGT to metric  $DTx$ . Let  $Sx^-$  be an endgame strategy minimising  $dtx$ , e.g., ‘quickest mate’  $SM^-$ ,  $SC^-$ ,  $SZ^-$  or  $SZ_{50}^-$ . Let  $Sx^+$  be a strategy maximising  $dtx$ . With some exceptions, q.v. Section 5.2,  $Sx^-$  strategies are used by attackers and  $Sx^+$  strategies are used by defenders.

Let  $SZ^\circ$  and  $SZ_k^\circ$  be endgame strategies *guarding* the length of the current phase in the context of a  $k$ -move rule and a remaining  $mleft$  moves before a possible draw claim. By definition, if  $dtx > mleft$ ,  $Sx^\circ \equiv Sx^-$ .

Some elementary observations are worth noting first:

- $Sx$  must not filter out all available moves, hence the contingency definition of  $Sx^\circ$ ,
- $Sxy$  defines at least as narrow a choice of moves as  $Sx$ ,
- if  $Sxy$  fails to safeguard the theoretical value of the position, then  $Sx$  also fails,
- if  $Sy$  has no effect after the use of  $Sx$ , then  $Sxy \equiv Sx$ ,
- $SZ_k^\circ$  has no effect if the position is a draw under the  $k$ -move rule
- $Sxx \equiv Sx$ , i.e. a strategy ‘filter’ has no further effect when applied a 2nd time,
- $Sxy$  is not necessarily identical to  $Syx$ , e.g.,  $SM^-Z^-$  and  $SZ^-M^-$  are different,
- $Sxy \equiv Sx \equiv Syx$  if  $Sx$  excludes any move that  $Sy$  excludes,
- $SZ^\circ Z^- \equiv SZ^-$ :  $SZ^\circ$  allows  $DTZ$ -optimal moves through its filter in all positions.

A likely set of goals for an attacking endgame strategy is to:

- win from any position that can be won under the prevailing  $k$ -move rule,
- avoid a draw-claim in the current phase if possible, and
- maximize the probability of finessing a win from a draw against a fallible player.

<sup>9</sup> In fact, intelligent access-code interpreting ‘ $DTZ_{50} > 50$ ’ as “draw” enables this  $EdZ_{50}Z$  encoding:

“ $DTZ > 50 \vee EZ$  code =  $EZ_k$  code”  $\Rightarrow$   $EdZ_{50}Z$  stores 0 (reducing, e.g.,  $KRNKNN EdZ_{50}Z$  to *null*).

“ $DTZ \leq 50$  but new  $EZ_{50}$  draw”  $\Rightarrow$   $EdZ_{50}Z$  stores 1. “ $0 < DTZ_{50} - DTZ = \delta$ ”  $\Rightarrow$   $EdZ_{50}Z$  stores  $\delta+1$ .

It is already clear from KBBKP, KNNKP, KQPKQ and KRPKP examples [18] that the three strategies  $SC^-$ ,  $SM^-$  and  $SZ^-$ , even in combination, are not enough to achieve even the first goal. As conjectured by Haworth [3], and demonstrated by Bourzutschky [2], KBBKNN includes positions where these three strategies all fail, not even including the move which safeguards a win available under the current 50-move rule. Similar positions have been found in KBBKBN, KQNKBB and KBNNKQ by Tamplin and their strategy-driven lines are illustrated in Appendix 1 after Table 6. However, the first objective is in fact relatively easy:  $SZ_k^-$  wins any position winnable against best play under a  $k$ -move rule. As  $k$  is currently 50,  $DTZ_{50}$  EGTs and  $SZ_{50}^-$  have a clear role. The strategy  $SZ_{50}^-$  provides no help in other situations where finesse and/or the opponent's acquiescence are required: more sophisticated strategies are required.

### 5.1 Strategies for playing a fallible opponent

By definition, a *fallible* opponent is not certain to achieve a result as good as the theoretical value of the position. They may lose a half or full point, fail to avoid a 50-move draw claim from the opponent or fail to defend a lost position long enough to claim an available draw. Let us suppose that it is possible to avoid a draw-claim in the current phase, if not in a later phase. Clearly, it is critical to achieve this if a win is to follow.

The strategy  $SZ^-$  does so but strives for nothing else. The strategy  $SZ^\circ Z_k^-$  does so, and also seizes on any winnable position once offered. The strategy  $SZ^\circ Z_k^- Z^-$  also achieves a third, ancillary goal of achieving both goals in the shortest current phase.  $SZ^\circ Z_k^- Z^-$  is not however the best use of  $DTZ$  and  $DTZ_k$  data. It does not attempt to minimize the difficulty of finessing the win in the second and subsequent phases of play. In particular, the third goal runs counter to giving the fallible opponent the best opportunity to concede ground in the current phase.

To increase the chance of finessing a win against a fallible opponent, it is helpful to play the opponent as well as the game by exploiting any apparent fallibility [5,6,9]. This is done by having an *opponent model* OM, e.g.,  $R_c$  [5], and using it in a forward search. As the opponent's fallibility replaces certainty by probabilities, the forward search minimaxes expected depth rather than depth. The OM may be revised by a Bayesian learning process in the light of experience during play.

### 5.2 Winning under a $k$ -move rule

The underlying difficulty is that the data so far does not help us to answer the question "By how much does the current position fail to be a win under the 50-move rule?". However, the question implicitly defines a new metric:

$dtr$  = the least  $k$  for which a position is won or lost, given a  $k$ -move drawing rule,

$0 \leq dtr \leq dtm$  and therefore the integer  $dtr$  can be determined.  $dtr-k$  measures the defender's margin for error and the attacker's challenge when there are  $k$  moves left before a draw-claim in the current phase. Although the 50-move rule seems unlikely to be changed to a different  $k$ -move rule, the DTR EGT enables an attacker to win any position winnable under any  $k$ -move rule, regardless of  $k$ . It obviates the need for specific  $DTZ_{50}$  EGTs.

Because a sequence of positions on the winning line can have the same DTR value, the following metric is also necessary [4] while generating and using the DTR EGTs:

$dtz_R$  = the minimaxed depth to a (move-count zeroing) move while minimaxing  $dtr$

$SZ^{\circ}R^{\circ}Z_R^{-}$  is a necessary and sufficient strategy to achieve any win available against best play given a  $k$ -move rule.  $SR^+Z_R^+$  is a necessary and sufficient strategy to defend a  $k$ -move draw.

Generating the DTR EGTs remains a future challenge, made the more difficult because two metrics are used in parallel, and the process is not as efficient as that for DTC, DTM and potentially DTZ. However, because  $dtr \geq dtz_R \geq dtz$ ,  $dtz_R$  and  $dtr$  may be derived economically from tables EZ, EdZ<sub>R</sub>Z and EdRZ<sub>R</sub> in the same way<sup>10</sup> as  $dtz_{50}$  is derived from tables EZ and EdZ<sub>50</sub>Z.

The  $SZ^{\circ}R^{\circ}Z_R^{-}$  strategy minimizes DTR, but only within the constraints of completing the current phase in the available moves and without forward search. It might therefore require too many moves to retain a target  $dtr$  to the end of the phase.

With the addition of the  $SZ_R^{\circ}$  filter, strategy  $SZ^{\circ}Z_R^{\circ}R^{\circ}Z_R^{-}$  aims to adopt an in-range DTR goal to ameliorate this problem. It:

- guards the length of the current phase in the context of the current  $k$ -move rule,
- wins any position that is winnable under whatever  $k$ -move rule is in force,
- aims to minimize  $dtr$  for the attacking side with pragmatic DTR goals, and
- achieves the first three goals in a current phase of least possible moves.

Similar caveats apply to  $SZ^{\circ}Z_R^{\circ}R^{\circ}Z_R^{-}$  as to  $SZ^{\circ}Z_k^{\circ}Z^{-}$ . The strategy does not necessarily minimize DTR, or  $\bar{R}$  = Expected[DTR] against a fallible opponent. It does not even make best use of the moves available to give the opponent more opportunity to err. Within constraints which avoid 3x repetition<sup>11</sup>, a more liberal strategy such as  $SZ^{\circ}Z_R^{\circ}R^{\circ}Z_R^+$  can be more effective than  $SZ^{\circ}Z_R^{\circ}R^{\circ}Z_R^{-}$ . In position NN-P<sup>12</sup>,  $SZ^{\circ}Z_R^{\circ}R^{\circ}Z_R^{-}$  makes the optimal move-choice<sup>13</sup> Nb1+:  $SZ^{\circ}Z_{50}^{-}$  can, and  $S\sigma$  ( $\sigma \equiv C^{-}, M^{-}, Z^{-}, Z^{\circ}Z_{50}^{-}Z^{-}$ ) do, concede DTR depth with Kc2.

### 5.3 Strategy effectiveness

The effectiveness of an attacking strategy may be measured in two dimensions:

- % of theoretically won positions in which the strategy retains the win  
i.e. in which the strategy offers no moves which are not offered by  $SZ_{50}^{-}$
- % of drawn positions in which a win is finessed against a fallible opponent

Different reference defenders are needed for the two dimensions. We suggest here:

- for a lost position, an infallible defender playing strategy  $SR^+Z_R^+$ , and otherwise,
- a fallible defender  $R_c$  [6] playing ‘to’ DTR and DTZ<sub>R</sub>.

<sup>10</sup> Because there are no ‘extra’ draws as in EdZ<sub>50</sub>Z, EdZ<sub>R</sub>Z  $\equiv \{dtz_R - dtz\}$  and EdRZ<sub>R</sub>  $\equiv \{dtr - dtz_R\}$ .

<sup>11</sup> e.g., sufficient but not necessary, no  $\{DTR, DTZ_R\}$  combination to be visited three times.

<sup>12</sup> NN-P: 8/8/2pN4/8/k1N5/8/2K5 w.  $dtr=115p$ ,  $dtr=102p$ ,  $dtz=42p$ ,  $dtz_R=60p$ .

<sup>13</sup>  $SZ_R^{\circ}R^{\circ}Z_R^{-}$  -  $SR^+Z_R^+$ : 1. Nb1+! Ka4!. White retains DTR = 102p and converts in 30m.

In the context of the 50-move rule,  $SZ_{50}$  retains the win in 100% of positions. Although this has not been examined, we expect  $SZ$ ,  $SC$ ,  $SMC$  and  $SM$  to exhibit increasing rates of failure.  $SZ$  fails both in the 0.34% of positions where  $DTZ < DTZ_{50}$  and in positions with  $DTZ = DTZ_{50}$  where it offers moves which  $SZ_{50}$  rejects.<sup>14</sup>

## 6 EGT integrity

All EGT files were immediately given MD5sum signatures [11] to guard against subsequent corruption or loss<sup>15</sup>. The EGTs were checked for errors in various ways:

- DTx EGTs  $\{Ex\}$ ,  $x = Z$  and  $Z_{50}$ , verified by Nalimov's standard test.
- consistency of the  $\{EM\}$  and  $\{EZ\}$  EGTs confirmed:  
counts of all positions found identical to predicted index-ranges, and theoretical values found identical with  $dtm \geq dtz$ .
- consistency of the  $\{EZ_{50}\}$  and  $\{EZ\}$  EGTs confirmed:  
values identical with  $dtz_{50} \geq dtz$ , or 'EZ' win/loss an 'EZ<sub>50</sub>' draw,
- DTZ statistics compared with Stiller's results [14],
- published DTZ-minimaxing lines [14] checked against DTZ EGTs, and
- DTZ statistics compared with Thompson's results [17].

Multi-metric working introduces new risks to the process of EGT generation and we recommend that the EGTs are self-identifying to increase integrity assurance.

## 7 Summary

This paper is a second snapshot of continuing work on the evolution and use of a multi-metric code 'NBT'. This was created by Nalimov, generalized by Bourzutschky [2] and managed on Unix by Tamplin. Here, we surveyed the newly completed 6-man pawnless DTZ and  $DTZ_{50}$  data. The 3-6-man pawnless DTZ EGTs  $\{EZ\}$  to date are 56.17% the size of the equivalent set  $\{EM\}$  and the compressed  $EdZ_{50}Z$  EGTs increase this figure to 57.28%. These percentages will reduce as the 6-man P-endgame and 5-1 pawnless EGTs are generated. This is an attractive, practical benefit as the 3-to-6-man EMs will be some 1.45 TB in size.

Clearly, there are more effective and efficient endgame strategies than the commonly used  $SM$ , and the only constraint is access to EGTs. It is recommended that  $SC^M$ ,  $SZ^M Z$ ,  $SZ^o Z_{50} Z$  and perhaps other strategies are considered, and that the EC, EZ and  $EdZ_{50}Z$  EGTs are made available to enable their use. The computation of DTR and  $DTZ_R$  EGTs remains a future challenge. Endgame strategies related to  $SZ^o Z_R^o R^Z Z_R$  promise to remove many of the chessic artificialities induced by current metric-based strategies, such as DTZ-motivated sacrifices by the attacker and incorrect choices of defensive goal by the losing side.

<sup>14</sup> e.g., 7K/8/3q4/3B4/5Nk1/8/3B4/8 b:  $DTZ = DTZ_{50} = 13$  but  $SZ$  allows Qc7 leading to a 50m-draw.

<sup>15</sup> An invaluable guard which enabled the successful recovery of almost all the 0.6TB of EGT data at risk after a RAID crash in the last stages of production work for this paper.

## Acknowledgements

We thank Eugene Nalimov for two versions of his code, the 2001 version which the first author evolved to multi-metric form, and the 2003 version. Marc Bourzutschky also championed the merits of DTZ<sub>50</sub> in the absence of DTR data, and contributed several major computations. We thank Rafael Andrist [1] for a ‘multi-metric’ WILHELM to data-mine the EGTs, and Bob Hyatt for occasional help.

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## Appendix: Chess Endgame Data and Examples

Table 3a. Chess Endgames: 6-man, pawnless DTC/Z data.<sup>16</sup>

| Endgame |         |     | DTC Metric            |       |       |        |               |     |     |     |
|---------|---------|-----|-----------------------|-------|-------|--------|---------------|-----|-----|-----|
|         |         |     | # of maxDTC positions |       |       |        | maxDTC, moves |     |     |     |
|         |         |     | 1-0                   |       | 0-1   |        | 1-0           |     | 0-1 |     |
| Endgame | GBR     | w-b | wtm                   | btm   | wtm   | btm    | wtm           | btm | wtm | btm |
| KBBKBB  | 0080.00 | 3-3 | 704                   | 224   | 224   | 704    | 6             | 5   | 5   | 6   |
| KBBKBN  | 0053.00 | 3-3 | 10                    | 2     | 26    | 180    | 28            | 27  | 9   | 10  |
| KBBKNN  | 0026.00 | 3-3 | 11                    | 1     | 488   | 1,518  | 38            | 38  | 3   | 4   |
| KBNKBN  | 0044.00 | 3-3 | 29                    | 4     | 4     | 29     | 9             | 8   | 8   | 9   |
| KBNKNN  | 0017.00 | 3-3 | 1                     | 1     | 12    | 154    | 13            | 12  | 6   | 7   |
| KNNKNN  | 0008.00 | 3-3 | 44                    | 8     | 8     | 44     | 7             | 6   | 6   | 7   |
| KQBKBB  | 1070.00 | 3-3 | 3                     | 13    | 1,317 | 6,118  | 13            | 13  | 3   | 4   |
| KQBKBN  | 1043.00 | 3-3 | 13                    | 107   | 944   | 4,097  | 16            | 16  | 3   | 4   |
| KQBKNN  | 1016.00 | 3-3 | 71                    | 331   | 28    | 81     | 13            | 13  | 2   | 3   |
| KQBKQB  | 4040.00 | 3-3 | 2                     | 3     | 3     | 2      | 46            | 45  | 45  | 46  |
| KQBKQN  | 4013.00 | 3-3 | 2                     | 1     | 3     | 15     | 36            | 36  | 32  | 32  |
| KQBKRB  | 1340.00 | 3-3 | 2                     | 11    | 15    | 30     | 42            | 41  | 6   | 7   |
| KQBKRN  | 1313.00 | 3-3 | 1                     | 6     | 9     | 34     | 27            | 27  | 7   | 8   |
| KQBKRR  | 1610.00 | 3-3 | 1                     | 79    | 21    | 23     | 85            | 84  | 10  | 11  |
| KQNKBB  | 1061.00 | 3-3 | 8                     | 32    | 1,521 | 6,573  | 15            | 15  | 3   | 4   |
| KQNKBN  | 1034.00 | 3-3 | 1                     | 7     | 3     | 3      | 17            | 17  | 4   | 5   |
| KQNKNN  | 1007.00 | 3-3 | 27                    | 137   | 74    | 207    | 16            | 16  | 2   | 3   |
| KQNKQN  | 4004.00 | 3-3 | 6                     | 2     | 2     | 6      | 29            | 29  | 29  | 29  |
| KQNKRB  | 1331.00 | 3-3 | 11                    | 26    | 8     | 20     | 26            | 26  | 8   | 9   |
| KQNKRN  | 1304.00 | 3-3 | 1                     | 1     | 2     | 11     | 40            | 40  | 9   | 9   |
| KQNKRR  | 1601.00 | 3-3 | 7                     | 6     | 6     | 7      | 152           | 152 | 11  | 12  |
| KQOKBB  | 2060.00 | 3-3 | 984                   | 5,128 | 137   | 714    | 6             | 6   | 3   | 4   |
| KQOKBN  | 2033.00 | 3-3 | 4                     | 28    | 99    | 376    | 8             | 8   | 3   | 4   |
| KQOKNN  | 2006.00 | 3-3 | 2                     | 8     | 1     | 36,110 | 7             | 7   | 1   | 1   |
| KQOKQB  | 5030.00 | 3-3 | 8                     | 1     | 1     | 2      | 62            | 62  | 22  | 23  |
| KQOKQN  | 5003.00 | 3-3 | 4                     | 26    | 4     | 20     | 50            | 50  | 18  | 19  |
| KQOKQQ  | 8000.00 | 3-3 | 1                     | 2     | 2     | 1      | 44            | 44  | 44  | 44  |
| KQOKQR  | 5300.00 | 3-3 | 4                     | 2     | 1     | 12     | 48            | 47  | 56  | 56  |
| KQOKRB  | 2330.00 | 3-3 | 4                     | 22    | 21    | 26     | 14            | 13  | 5   | 6   |
| KQOKRN  | 2303.00 | 3-3 | 2                     | 12    | 14    | 11     | 14            | 14  | 5   | 6   |
| KQOKRR  | 2600.00 | 3-3 | 3                     | 7     | 483   | 575    | 18            | 18  | 5   | 6   |
| KQRKBB  | 1160.00 | 3-3 | 3                     | 13    | 689   | 3,514  | 12            | 12  | 3   | 4   |
| KQRKBN  | 1133.00 | 3-3 | 3                     | 14    | 419   | 1,645  | 11            | 11  | 3   | 4   |
| KQRKNN  | 1106.00 | 3-3 | 1                     | 243   | 20    | 40     | 11            | 10  | 2   | 3   |
| KQRKQB  | 4130.00 | 3-3 | 2                     | 12    | 5     | 3      | 73            | 73  | 31  | 32  |
| KQRKQN  | 4103.00 | 3-3 | 3                     | 4     | 2     | 6      | 71            | 71  | 26  | 27  |
| KQRKQR  | 4400.00 | 3-3 | 3                     | 1     | 1     | 3      | 92            | 92  | 92  | 92  |
| KQRKRB  | 1430.00 | 3-3 | 2                     | 10    | 75    | 92     | 21            | 21  | 5   | 6   |
| KQRKRN  | 1403.00 | 3-3 | 1                     | 7     | 8     | 16     | 21            | 21  | 6   | 7   |
| KQRKRR  | 1700.00 | 3-3 | 6                     | 4     | 2     | 8      | 34            | 34  | 10  | 11  |
| KRBKBB  | 0170.00 | 3-3 | 14                    | 3     | 97    | 252    | 83            | 83  | 5   | 6   |
| KRBKBN  | 0143.00 | 3-3 | 1                     | 6     | 1     | 9      | 98            | 98  | 5   | 6   |
| KRBKNN  | 0116.00 | 3-3 | 1                     | 2     | 82    | 196    | 223           | 222 | 2   | 3   |
| KRBKRB  | 0440.00 | 3-3 | 5                     | 1     | 1     | 5      | 17            | 16  | 16  | 17  |
| KRBKRN  | 0413.00 | 3-3 | 78                    | 45    | 2     | 25     | 21            | 20  | 13  | 14  |
| KRNKBB  | 0161.00 | 3-3 | 13                    | 14    | 4     | 20     | 140           | 140 | 9   | 10  |
| KRNKBN  | 0134.00 | 3-3 | 1                     | 7     | 12    | 36     | 190           | 189 | 5   | 6   |

<sup>16</sup> The ‘GBR’ code, created by Guy, Blandford and Roycroft, associates the endgame force with a number of form qrbn.(w)p(b)p, assigning ‘1’ to White’s men and ‘3’ to Black’s. Thus KQNKRB  $\equiv$  1331.00.

A ‘9’ indicates more than two like pieces of a colour. Thus, KBBBKB  $\equiv$  0090.00/31.

Table 3b. Chess Endgames: 6-man, pawnless DTC/Z data.

|         |            |     | DTC Metric            |           |         |           |               |     |     |     |
|---------|------------|-----|-----------------------|-----------|---------|-----------|---------------|-----|-----|-----|
| Endgame |            |     | # of maxDTC positions |           |         |           | maxDTC, moves |     |     |     |
|         |            |     | 1-0                   |           | 0-1     |           | 1-0           |     | 0-1 |     |
| Endgame | GBR        | w-b | wtm                   | btm       | wtm     | btm       | wtm           | btm | wtm | btm |
| KRNKNN  | 0107.00    | 3-3 | 1                     | 7         | 29      | 54        | 243           | 242 | 3   | 4   |
| KRNKRN  | 0404.00    | 3-3 | 6                     | 3         | 3       | 6         | 21            | 20  | 20  | 21  |
| KRRKBB  | 0260.00    | 3-3 | 2                     | 16        | 1       | 4         | 37            | 37  | 4   | 5   |
| KRRKBN  | 0233.00    | 3-3 | 3                     | 42        | 6       | 30        | 26            | 25  | 4   | 5   |
| KRRKNN  | 0206.00    | 3-3 | 2                     | 3         | 37      | 77        | 33            | 33  | 2   | 3   |
| KRRKRB  | 0530.00    | 3-3 | 22                    | 13        | 1       | 455       | 54            | 54  | 6   | 6   |
| KRRKRN  | 0503.00    | 3-3 | 2                     | 3         | 37      | 89        | 73            | 73  | 6   | 7   |
| KRRKRR  | 0800.00    | 3-3 | 2                     | 3         | 3       | 2         | 18            | 17  | 17  | 18  |
| KBBBKB  | 0090.00/31 | 4-2 | 19                    | 6         | 6,150   | 21,903    | 20            | 20  | 1   | 2   |
| KBBBKN  | 0093.00/30 | 4-2 | 6                     | 6         | 951     | 4,838     | 12            | 12  | 0   | 1   |
| KBBBKQ  | 1090.00/30 | 4-2 | 1                     | 9         | 1       | 3         | 10            | 9   | 51  | 51  |
| KBBBKR  | 0390.00/30 | 4-2 | 1                     | 23        | 13      | 72        | 69            | 68  | 4   | 5   |
| KBBNKB  | 0051.00    | 4-2 | 3                     | 4         | 10,340  | 38,254    | 36            | 36  | 1   | 2   |
| KBBNKN  | 0024.00    | 4-2 | 9                     | 54        | 3,663   | 18,984    | 31            | 31  | 0   | 1   |
| KBBNKQ  | 3021.00    | 4-2 | 122                   | 16        | 17      | 1         | 12            | 11  | 62  | 63  |
| KBBNKR  | 0321.00    | 4-2 | 4                     | 2         | 10      | 50        | 68            | 68  | 6   | 7   |
| KBNNKB  | 0042.00    | 4-2 | 6                     | 4         | 4,779   | 18,249    | 38            | 38  | 1   | 2   |
| KBNNKN  | 0025.00    | 4-2 | 17                    | 56        | 4,335   | 22,890    | 28            | 28  | 0   | 1   |
| KBNNKQ  | 3012.00    | 4-2 | 5                     | 1         | 1       | 4         | 12            | 11  | 49  | 49  |
| KBNNKR  | 0312.00    | 4-2 | 12                    | 4         | 1       | 398       | 49            | 48  | 7   | 7   |
| KNNNKB  | 0039.00/30 | 4-2 | 1                     | 2         | 1,275   | 2,891     | 92            | 91  | 0   | 1   |
| KNNNKN  | 0009.00/31 | 4-2 | 2                     | 2         | 1,584   | 8,562     | 86            | 86  | 0   | 1   |
| KNNNKQ  | 1009.00/30 | 4-2 | 1                     | 1         | 6       | 11        | 9             | 8   | 35  | 35  |
| KNNNKR  | 0309.00/30 | 4-2 | 2                     | 2         | 8       | 31        | 12            | 11  | 6   | 7   |
| KQBBKB  | 1050.00    | 4-2 | 221                   | 1,027     | 9,168   | 34,389    | 8             | 8   | 1   | 2   |
| KQBBKN  | 1023.00    | 4-2 | 122                   | 515       | 1,327   | 6,813     | 7             | 7   | 0   | 1   |
| KQBBKQ  | 4020.00    | 4-2 | 1                     | 1         | 2       | 3         | 93            | 93  | 15  | 16  |
| KQBBKR  | 1320.00    | 4-2 | 2                     | 12        | 146,288 | 830,146   | 20            | 20  | 1   | 2   |
| KQBNKB  | 1041.00    | 4-2 | 28                    | 191       | 7,873   | 31,019    | 7             | 7   | 1   | 2   |
| KQBNKN  | 1014.00    | 4-2 | 133                   | 708       | 3,262   | 17,347    | 6             | 6   | 0   | 1   |
| KQBNKQ  | 4011.00    | 4-2 | 1                     | 1         | 1       | 1         | 65            | 65  | 16  | 17  |
| KQBNKR  | 1311.00    | 4-2 | 4                     | 28        | 408,029 | 2,319,030 | 22            | 22  | 1   | 2   |
| KQNNKB  | 1032.00    | 4-2 | 3                     | 21        | 1,457   | 3,516     | 11            | 11  | 0   | 1   |
| KQNNKN  | 1005.00    | 4-2 | 7                     | 21        | 1,806   | 9,962     | 9             | 9   | 0   | 1   |
| KQNNKQ  | 4002.00    | 4-2 | 2                     | 2         | 5       | 20        | 71            | 71  | 13  | 14  |
| KQNNKR  | 1302.00    | 4-2 | 2                     | 12        | 25      | 163       | 22            | 22  | 2   | 3   |
| KQOBKB  | 2040.00    | 4-2 | 2                     | 10        | 1,665   | 7,712     | 5             | 5   | 1   | 2   |
| KQOBKN  | 2013.00    | 4-2 | 23                    | 130       | 440     | 2,285     | 5             | 5   | 0   | 1   |
| KQOBKQ  | 5010.00    | 4-2 | 6                     | 30        | 7       | 23        | 29            | 29  | 9   | 10  |
| KQOBKR  | 2310.00    | 4-2 | 1                     | 5         | 75,802  | 478,709   | 26            | 26  | 1   | 2   |
| KQONKB  | 2031.00    | 4-2 | 9,757                 | 37,511    | 383     | 1,005     | 4             | 4   | 0   | 1   |
| KQONKN  | 2004.00    | 4-2 | 49                    | 260       | 477     | 2,700     | 5             | 5   | 0   | 1   |
| KQONKQ  | 5001.00    | 4-2 | 1                     | 1         | 2       | 13        | 28            | 28  | 8   | 9   |
| KQONKR  | 1301.00    | 4-2 | 2                     | 12        | 112,955 | 720,034   | 24            | 24  | 1   | 2   |
| KQOQKB  | 9030.00/30 | 4-2 | 673,004               | 2,775,033 | 0       | 0         | 3             | 3   | —   | —   |
| KQOQKN  | 9003.00/30 | 4-2 | 827                   | 4,016     | 0       | 0         | 4             | 4   | —   | —   |
| KQOQKQ  | 9000.00/31 | 4-2 | 6                     | 40        | 1       | 5         | 19            | 19  | 9   | 10  |
| KQOQKR  | 9300.00/30 | 4-2 | 3                     | 19        | 11,025  | 77,175    | 20            | 20  | 1   | 2   |
| KQORKB  | 2130.00    | 4-2 | 438                   | 1,766     | 0       | 0         | 4             | 4   | —   | —   |
| KQORKN  | 2103.00    | 4-2 | 5                     | 29        | 572     | 2,459     | 5             | 5   | 0   | 1   |
| KQORKQ  | 5100.00    | 4-2 | 3                     | 7         | 3       | 13        | 28            | 28  | 9   | 10  |
| KQORKR  | 2400.00    | 4-2 | 1                     | 7         | 63,979  | 447,853   | 24            | 24  | 1   | 2   |
| KQRBKB  | 1140.00    | 4-2 | 83                    | 415       | 5,158   | 23,146    | 5             | 5   | 1   | 2   |

Table 3c. Chess Endgames: 6-man, pawnless DTC/Z data.

| Endgame |            |     | DTC Metric            |           |         |           |               |     |     |     |
|---------|------------|-----|-----------------------|-----------|---------|-----------|---------------|-----|-----|-----|
|         |            |     | # of maxDTC positions |           |         |           | maxDTC, moves |     |     |     |
|         |            |     | 1-0                   |           | 0-1     |           | 1-0           |     | 0-1 |     |
| Endgame | GBR        | w-b | wtm                   | btm       | wtm     | btm       | wtm           | btm | wtm | btm |
| KORBKN  | 1113.00    | 4-2 | 977                   | 4,872     | 3,390   | 15,732    | 5             | 5   | 0   | 1   |
| KORBKQ  | 4110.00    | 4-2 | 6                     | 19        | 4       | 9         | 49            | 49  | 12  | 13  |
| KORBKR  | 1410.00    | 4-2 | 1                     | 7         | 269,633 | 1,690,187 | 25            | 25  | 1   | 2   |
| KORNKB  | 1131.00    | 4-2 | 1,358,087             | 5,054,177 | 1,150   | 2,838     | 4             | 4   | 0   | 1   |
| KORNKN  | 1104.00    | 4-2 | 12                    | 76        | 3,450   | 16,495    | 6             | 6   | 0   | 1   |
| KORNKQ  | 4101.00    | 4-2 | 3                     | 7         | 1       | 3         | 55            | 55  | 11  | 12  |
| KORNKR  | 1401.00    | 4-2 | 2                     | 14        | 375,359 | 2,375,039 | 24            | 24  | 1   | 2   |
| KORRKB  | 1230.00    | 4-2 | 74,085                | 294,223   | 0       | 0         | 4             | 4   | —   | —   |
| KORRKN  | 1203.00    | 4-2 | 299                   | 1,474     | 1,498   | 6,333     | 5             | 5   | 0   | 1   |
| KORRKQ  | 4200.00    | 4-2 | 1                     | 4         | 1       | 2         | 41            | 41  | 8   | 9   |
| KORRRK  | 1500.00    | 4-2 | 12                    | 82        | 115,042 | 805,294   | 23            | 23  | 1   | 2   |
| KRBBKB  | 0150.00    | 4-2 | 4                     | 13        | 12,789  | 47,143    | 18            | 18  | 1   | 2   |
| KRBBKN  | 0123.00    | 4-2 | 7                     | 57        | 3,717   | 17,552    | 12            | 11  | 0   | 1   |
| KRBBKQ  | 3120.00    | 4-2 | 3                     | 1         | 2       | 1         | 44            | 44  | 25  | 26  |
| KRBBKR  | 0420.00    | 4-2 | 1                     | 27        | 104     | 787       | 36            | 35  | 2   | 3   |
| KRBNKB  | 0141.00    | 4-2 | 9                     | 2         | 10,985  | 42,661    | 13            | 13  | 1   | 2   |
| KRBNKN  | 0114.00    | 4-2 | 1                     | 6         | 8,152   | 39,422    | 12            | 12  | 0   | 1   |
| KRBNKQ  | 3111.00    | 4-2 | 4                     | 3         | 3       | 1         | 99            | 98  | 28  | 29  |
| KRBNKR  | 0411.00    | 4-2 | 1                     | 1         | 9       | 55        | 36            | 36  | 3   | 4   |
| KRNNKB  | 0132.00    | 4-2 | 31                    | 44        | 2,094   | 4,814     | 12            | 12  | 0   | 1   |
| KRNNKN  | 0105.00    | 4-2 | 154                   | 2,477     | 4,138   | 20,608    | 13            | 12  | 0   | 1   |
| KRNNKQ  | 3102.00    | 4-2 | 2                     | 1         | 2       | 3         | 28            | 27  | 41  | 41  |
| KRNNKR  | 0402.00    | 4-2 | 1                     | 3         | 28      | 114       | 39            | 39  | 3   | 4   |
| KRRBKB  | 0240.00    | 4-2 | 530                   | 1,911     | 3,931   | 17,132    | 7             | 7   | 1   | 2   |
| KRRBKN  | 0213.00    | 4-2 | 2,459                 | 12,709    | 3,664   | 16,427    | 6             | 6   | 0   | 1   |
| KRRBKQ  | 3210.00    | 4-2 | 3                     | 4         | 2       | 5         | 82            | 82  | 16  | 17  |
| KRRBKR  | 0510.00    | 4-2 | 2                     | 10        | 221,774 | 1,375,964 | 31            | 31  | 1   | 2   |
| KRRNKB  | 0231.00    | 4-2 | 716                   | 2,439     | 825     | 1,937     | 7             | 7   | 0   | 1   |
| KRRNKN  | 0204.00    | 4-2 | 69                    | 333       | 3,537   | 16,109    | 7             | 7   | 0   | 1   |
| KRRNKQ  | 3201.00    | 4-2 | 5                     | 2         | 2       | 1         | 101           | 101 | 18  | 19  |
| KRRNKR  | 0501.00    | 4-2 | 9                     | 46        | 289,032 | 1,811,539 | 33            | 33  | 1   | 2   |
| KRRRKB  | 0930.00/30 | 4-2 | 51,108                | 219,810   | 0       | 0         | 5             | 5   | —   | —   |
| KRRRKN  | 0903.00/30 | 4-2 | 6                     | 30        | 950     | 3,965     | 6             | 6   | 0   | 1   |
| KRRRKQ  | 3900.00/30 | 4-2 | 3                     | 5         | 1       | 2         | 65            | 65  | 13  | 14  |
| KRRRRK  | 0900.00/31 | 4-2 | 3                     | 6         | 64,686  | 452,802   | 21            | 21  | 1   | 2   |

Table 4a. Chess Endgames: 6-man, pawnless DTZ<sub>50</sub> data.

| Endgame |         |     | DTZ <sub>50</sub> Metric |         |       |       |                  |     |     |     |
|---------|---------|-----|--------------------------|---------|-------|-------|------------------|-----|-----|-----|
|         |         |     | # of maximal positions   |         |       |       | max depth, moves |     |     |     |
|         |         |     | 1-0                      |         | 0-1   |       | 1-0              |     | 0-1 |     |
| Endgame | GBR     | w-b | wtm                      | btm     | wtm   | btm   | wtm              | btm | wtm | btm |
| KBBKBN  | 0053.00 | 3-3 | 5                        | 1       | 26    | 180   | 21               | 20  | 9   | 10  |
| KBBKNN  | 0026.00 | 3-3 | 46                       | 17      | 488   | 1,518 | 29               | 28  | 3   | 4   |
| KBNKBN  | 0044.00 | 3-3 | 29                       | 4       | 4     | 29    | 9                | 8   | 8   | 9   |
| KBNKNN  | 0017.00 | 3-3 | 1                        | 1       | 12    | 154   | 13               | 12  | 6   | 7   |
| KQBKBB  | 1070.00 | 3-3 | 8                        | 30      | 1,317 | 6,118 | 13               | 13  | 3   | 4   |
| KQBKNN  | 1016.00 | 3-3 | 71                       | 331     | 28    | 81    | 13               | 13  | 2   | 3   |
| KQBKRR  | 1610.00 | 3-3 | 111,887                  | 251,377 | 21    | 23    | 50               | 50  | 10  | 11  |
| KQNKBB  | 1061.00 | 3-3 | 15                       | 61      | 1     | 6,826 | 15               | 15  | 4   | 4   |
| KQNKBN  | 1034.00 | 3-3 | 1                        | 7       | 3     | 3     | 17               | 17  | 4   | 5   |
| KQNKNN  | 1007.00 | 3-3 | 27                       | 137     | 74    | 207   | 16               | 16  | 2   | 3   |

Table 4b. Chess Endgames: 6-man, pawnless DTZ<sub>50</sub> data.

| Endgame |            | DTZ <sub>50</sub> Metric |            |            |         |                  |     |     |     |    |
|---------|------------|--------------------------|------------|------------|---------|------------------|-----|-----|-----|----|
|         |            | # of maximal positions   |            |            |         | max depth, moves |     |     |     |    |
|         |            | 1-0                      |            | 0-1        |         | 1-0              |     | 0-1 |     |    |
| GBR     | w-b        | wtm                      | btm        | wtm        | btm     | wtm              | btm | wtm | btm |    |
| KQNKRR  | 1601.00    | 3-3                      | 3,007,192  | 2,814,979  | 6       | 7                | 50  | 50  | 11  | 12 |
| KQOKBB  | 2060.00    | 3-3                      | 1          | 5          | 137     | 714              | 8   | 8   | 3   | 4  |
| KQOKNN  | 2006.00    | 3-3                      | 2          | 8          | 1       | 36,110           | 7   | 7   | 1   | 1  |
| KQOKQB  | 5030.00    | 3-3                      | 81         | 247        | 1       | 2                | 50  | 50  | 22  | 23 |
| KQOKQR  | 5300.00    | 3-3                      | 4          | 2          | 6       | 26               | 48  | 47  | 50  | 50 |
| KQRKBB  | 1160.00    | 3-3                      | 3          | 13         | 689     | 3,514            | 12  | 12  | 3   | 4  |
| KQRKNN  | 1106.00    | 3-3                      | 1          | 243        | 20      | 40               | 11  | 10  | 2   | 3  |
| KQRKQB  | 4130.00    | 3-3                      | 1,989      | 1,841      | 5       | 3                | 50  | 50  | 31  | 32 |
| KQRKQN  | 4103.00    | 3-3                      | 1,953      | 1,698      | 2       | 6                | 50  | 50  | 26  | 27 |
| KQRKQR  | 4400.00    | 3-3                      | 1,191      | 837        | 837     | 1,191            | 50  | 50  | 50  | 50 |
| KQRKRB  | 1430.00    | 3-3                      | 2          | 10         | 75      | 92               | 21  | 21  | 5   | 6  |
| KRBKBB  | 0170.00    | 3-3                      | 69,308     | 36,223     | 97      | 252              | 50  | 50  | 5   | 6  |
| KRBKBN  | 0143.00    | 3-3                      | 12,633,808 | 15,861,502 | 1       | 9                | 50  | 50  | 5   | 6  |
| KRBKNN  | 0116.00    | 3-3                      | 1,944,494  | 2,800,448  | 82      | 196              | 50  | 50  | 2   | 3  |
| KRBKRB  | 0440.00    | 3-3                      | 5          | 1          | 1       | 5                | 17  | 16  | 16  | 17 |
| KRBKRN  | 0413.00    | 3-3                      | 78         | 45         | 2       | 25               | 21  | 20  | 13  | 14 |
| KRNKBB  | 0161.00    | 3-3                      | 2,037,618  | 1,042,171  | 4       | 20               | 50  | 50  | 9   | 10 |
| KRNKBN  | 0134.00    | 3-3                      | 2,488,599  | 1,948,808  | 13      | 38               | 50  | 50  | 5   | 6  |
| KRNKNN  | 0107.00    | 3-3                      | 1,202,592  | 1,198,532  | 29      | 54               | 50  | 50  | 3   | 4  |
| KRRKRB  | 0530.00    | 3-3                      | 372        | 107        | 1       | 455              | 50  | 50  | 6   | 6  |
| KRRKRN  | 0501.00    | 3-3                      | 4,335      | 3,898      | 37      | 89               | 50  | 50  | 6   | 7  |
| KBBBKN  | 0093.00/30 | 4-2                      | 3          | 6          | 951     | 4,838            | 14  | 14  | 0   | 1  |
| KBBBKQ  | 3090.00/30 | 4-2                      | 1          | 9          | 11      | 15               | 10  | 9   | 50  | 50 |
| KBBBKR  | 0390.00/30 | 4-2                      | 685,975    | 1,619,489  | 13      | 72               | 50  | 50  | 4   | 5  |
| KBBNKN  | 0024.00    | 4-2                      | 9          | 54         | 3,663   | 18,984           | 31  | 31  | 0   | 1  |
| KBBNKQ  | 3021.00    | 4-2                      | 122        | 16         | 8,148   | 4,176            | 12  | 11  | 50  | 50 |
| KBBNKR  | 0321.00    | 4-2                      | 139,436    | 248,016    | 10      | 50               | 50  | 50  | 6   | 7  |
| KBNNKN  | 0015.00    | 4-2                      | 3          | 3          | 4,335   | 22,890           | 29  | 29  | 0   | 1  |
| KBNNKQ  | 3012.00    | 4-2                      | 5          | 1          | 1       | 4                | 12  | 11  | 49  | 49 |
| KNNNKB  | 0039.00/30 | 4-2                      | 195,576    | 232,786    | 1,275   | 2,891            | 50  | 50  | 0   | 1  |
| KNNNKN  | 0009.00/31 | 4-2                      | 6,272      | 12,853     | 1,584   | 8,562            | 50  | 50  | 0   | 1  |
| KNNNKQ  | 3009.00/30 | 4-2                      | 1          | 1          | 6       | 11               | 9   | 8   | 35  | 35 |
| KQBBKN  | 1023.00    | 4-2                      | 122        | 515        | 1,327   | 6,813            | 7   | 7   | 0   | 1  |
| KQBBKQ  | 4020.00    | 4-2                      | 52,602     | 136,241    | 2       | 3                | 50  | 50  | 15  | 16 |
| KQBNKN  | 1014.00    | 4-2                      | 135        | 719        | 3,262   | 17,347           | 6   | 6   | 0   | 1  |
| KQBNKQ  | 4011.00    | 4-2                      | 297        | 885        | 1       | 1                | 50  | 50  | 16  | 17 |
| KQNNKQ  | 4002.00    | 4-2                      | 10,534     | 9,796      | 5       | 20               | 50  | 50  | 13  | 14 |
| KQORKQ  | 5100.00    | 4-2                      | 3          | 7          | 3       | 13               | 28  | 28  | 9   | 10 |
| KQRBKQ  | 41100.00   | 4-2                      | 6          | 19         | 4       | 9                | 49  | 49  | 12  | 13 |
| KQRBKR  | 1410.00    | 4-2                      | 1          | 7          | 269,633 | 1,690,187        | 25  | 25  | 1   | 2  |
| KQRNKQ  | 4101.00    | 4-2                      | 12         | 76         | 1       | 3                | 50  | 50  | 11  | 12 |
| KQRRKQ  | 4200.00    | 4-2                      | 1          | 4          | 1       | 2                | 41  | 41  | 8   | 9  |
| KRBBKN  | 0123.00    | 4-2                      | 7          | 57         | 3,717   | 17,552           | 12  | 11  | 0   | 1  |
| KRBBKQ  | 3120.00    | 4-2                      | 3          | 1          | 2       | 1                | 44  | 44  | 25  | 26 |
| KRBBKR  | 0420.00    | 4-2                      | 1          | 27         | 104     | 787              | 36  | 35  | 2   | 3  |
| KRBNKN  | 0114.00    | 4-2                      | 1          | 6          | 8,152   | 39,422           | 12  | 12  | 0   | 1  |
| KRBNKQ  | 3111.00    | 4-2                      | 120,325    | 34,369     | 3       | 1                | 50  | 50  | 28  | 29 |
| KRBNKR  | 0411.00    | 4-2                      | 1          | 1          | 9       | 55               | 36  | 36  | 3   | 4  |
| KRNNKQ  | 3102.00    | 4-2                      | 2          | 1          | 2       | 3                | 28  | 27  | 41  | 41 |
| KRRBKQ  | 3210.00    | 4-2                      | 23,857     | 56,552     | 2       | 5                | 50  | 50  | 16  | 17 |
| KRRBKR  | 0510.00    | 4-2                      | 2          | 10         | 221,774 | 1,375,964        | 31  | 31  | 1   | 2  |
| KRRNKQ  | 3201.00    | 4-2                      | 35,405     | 45,611     | 2       | 1                | 50  | 50  | 18  | 19 |
| KRRRKQ  | 3900.00/30 | 4-2                      | 271        | 1,195      | 1       | 2                | 50  | 50  | 13  | 14 |

Table 5a. The impact of the 50-move drawing rule on 6-man pawnless endgames.<sup>17</sup>

| Endgame |     | nominal wins  |             |             |             | % of nominal wins |       |         |       |
|---------|-----|---------------|-------------|-------------|-------------|-------------------|-------|---------|-------|
| res.    |     | # extra draws |             | # delayed   |             | extra draws       |       | delayed |       |
|         |     | wtm           | btm         | wtm         | btm         | wtm               | btm   | wtm     | btm   |
| KBBKBN  | 1-0 | 128,572,657   | 16,294,259  | 884,907     | 109,678     | 47.03             | 66.89 | 0.32    | 0.45  |
| KBBKNN  | 1-0 | 141,874,223   | 38,562,549  | 4,961,624   | 1,402,773   | 50.15             | 70.98 | 1.75    | 2.58  |
| KBNKBN  | 1-0 | 1,222,632     | 9,420       | 5,616       | 117         | 2.53              | 0.92  | 0.01    | 0.01  |
| KBNKNN  | 1-0 | 1,179,997     | 14,499      | 17,361      | 918         | 2.81              | 1.19  | 0.04    | 0.08  |
| KQBKBB  | 1-0 | 250,935       | 6,569,025   | 7,089,297   | 29,692,117  | 0.01              | 0.40  | 0.40    | 1.81  |
| KQBKNN  | 1-0 | 397           | 38,516      | 23,320      | 38,516      | ε                 | ε     | ε       | ε     |
| KQBKRR  | 1-0 | 586,397       | 1,305,447   | 0           | 0           | 0.04              | 0.16  | 0       | 0     |
| KQNKBB  | 1-0 | 300,774       | 6,546,430   | 11,971,950  | 45,591,146  | 0.02              | 0.41  | 0.64    | 2.84  |
| KQNKBN  | 0-1 | 6,167,236     | 125,922,828 | 17,522      | 259,838     | 69.89             | 47.75 | 0.20    | 0.10  |
|         | 0-1 | 3,703         | 1,213,657   | 26          | 1,328       | 1.05              | 2.80  | 0.01    | ε     |
| KQNKNN  | 1-0 | 188           | 36,110      | 59,575      | 242,663     | ε                 | ε     | ε       | 0.01  |
| KQNKRR  | 1-0 | 72,985,602    | 79,251,396  | 0           | 0           | 4.87              | 15.42 | 0       | 0     |
| KQQKBB  | 1-0 | 23,343        | 6,776,509   | 1,244,572   | 5,432,160   | ε                 | 0.58  | 0.18    | 0.47  |
| KQQKNN  | 1-0 | 130           | 44,687      | 4,704       | 22,000      | ε                 | ε     | ε       | ε     |
| KQQKQB  | 1-0 | 689           | 2,278       | 0           | 0           | ε                 | ε     | 0       | 0     |
| KQQKQR  | 0-1 | 17,313        | 41,775      | 42,552      | 66,504      | 0.02              | 0.01  | 0.04    | 0.01  |
| KQRKBB  | 1-0 | 125,901       | 6,357,673   | 2,948,393   | 11,781,268  | 0.01              | 0.37  | 0.18    | 0.69  |
| KQRKNN  | 1-0 | 249           | 39,230      | 9,116       | 46,469      | ε                 | ε     | ε       | ε     |
| KQRKQB  | 1-0 | 23,934        | 17,235      | 94,650      | 90,746      | ε                 | ε     | ε       | 0.01  |
| KQRKQN  | 1-0 | 12,641        | 11,010      | 70,821      | 86,758      | ε                 | ε     | ε       | 0.01  |
| KQRKQR  | 1-0 | 21,395        | 12,416      | 48,844      | 50,736      | ε                 | ε     | ε       | 0.01  |
| KQRKRB  | 0-1 | 251           | 11,459      | 3           | 410         | 0.01              | 0.02  | ε       | ε     |
| KRBKBB  | 1-0 | 2,561,991     | 1,304,230   | 0           | 0           | 0.22              | 0.68  | 0       | 0     |
| KRBKBN  | 1-0 | 426,514,269   | 767,645,636 | 0           | 0           | 12.14             | 41.47 | 0       | 0     |
| KRBKNN  | 1-0 | 331,894,421   | 676,322,987 | 0           | 0           | 16.18             | 45.17 | 0       | 0     |
| KRBKRB  | 1-0 | 9,084         | 783         | 1,605       | 122         | ε                 | ε     | ε       | ε     |
| KRBKRN  | 1-0 | 9,706         | 1,202       | 2,684       | 359         | ε                 | ε     | ε       | ε     |
| KRNKBB  | 1-0 | 407,078,847   | 370,216,259 | 0           | 0           | 26.20             | 66.08 | 0       | 0     |
| KRNKBN  | 0-1 | 13,836,487    | 133,053,338 | 117,223     | 640,177     | 65.00             | 47.50 | 0.55    | 0.23  |
|         | 1-0 | 139,761,310   | 107,975,414 | 0           | 0           | 4.98              | 15.04 | 0       | 0     |
| KRNKNN  | 0-1 | 9,921         | 1,225,920   | 316         | 6,092       | 1.12              | 2.53  | 0.04    | 0.01  |
|         | 1-0 | 82,794,630    | 83,586,263  | 0           | 0           | 5.18              | 14.78 | 0       | 0     |
| KRRKRB  | 1-0 | 380           | 145         | 0           | 0           | ε                 | ε     | 0       | 0     |
| KRRKRN  | 0-1 | 396           | 11,281      | 30          | 799         | 0.02              | 0.03  | ε       | ε     |
|         | 1-0 | 17,610        | 16,206      | 0           | 0           | ε                 | ε     | 0       | 0     |
| KBBBKN  | 1-0 | 743,762       | 37,035,833  | 55,589,963  | 161,070,140 | 0.15              | 6.16  | 11.28   | 26.80 |
| KBBBKQ  | 0-1 | 21,650,797    | 31,223,711  | 6,004,068   | 11,096,464  | 15.04             | 6.15  | 4.17    | 2.19  |
| KBBBKR  | 1-0 | 463,105       | 1,079,492   | 0           | 0           | 0.10              | 0.35  | 0       | 0     |
| KBBNKN  | 1-0 | 640,358       | 36,582,112  | 136,891,517 | 318,970,567 | 0.03              | 1.74  | 6.44    | 15.17 |
| KBBNKQ  | 0-1 | 55,226,710    | 40,880,784  | 27,763,565  | 27,296,005  | 10.16             | 2.52  | 5.11    | 1.68  |
| KBBNKR  | 1-0 | 184,213       | 312,436     | 0           | 0           | 0.01              | 0.05  | 0       | 0     |
| KBNNKN  | 1-0 | 96,123        | 1,016,653   | 10,322,215  | 13,062,956  | ε                 | 0.05  | 0.46    | 0.70  |
| KBNNKQ  | 0-1 | 178,774       | 178,631     | 179,015     | 143,015     | 0.03              | 0.01  | 0.03    | 0.01  |
| KNNNKB  | 1-0 | 539,360       | 648,931     | 0           | 0           | 0.08              | 0.20  | 0       | 0     |
| KNNNKN  | 1-0 | 86,880        | 154,950     | 0           | 0           | 0.01              | 0.03  | 0       | 0     |
| KNNNKQ  | 0-1 | 125,488       | 181,848     | 91,063      | 99,907      | 0.09              | 0.04  | 0.07    | 0.02  |
| KQBBKN  | 1-0 | 122,388       | 45,118,478  | 24,140,183  | 88,092,478  | 0.01              | 1.72  | 1.55    | 3.35  |
| KQBBKQ  | 1-0 | 206,322       | 526,510     | 0           | 0           | 0.01              | 0.05  | 0       | 0     |
|         | 0-1 | 413,225       | 39,206,954  | 96          | 4,608       | 12.21             | 8.59  | ε       | ε     |

<sup>17</sup> ‘ε’ indicates a non-zero value less than 0.005.

Table 5b. The impact of the 50-move drawing rule on 6-man pawnless endgames.

| Endgame |     | nominal wins  |            |            |            | % of nominal wins |      |         |      |
|---------|-----|---------------|------------|------------|------------|-------------------|------|---------|------|
|         |     | # extra draws |            | # delayed  |            | extra draws       |      | delayed |      |
|         |     | res.          | wtm        | btm        | wtm        | btm               | wtm  | btm     | wtm  |
| KQBNKN  | 1-0 | 38,709        | 1,197,026  | 852,368    | 2,263,825  | ε                 | 0.02 | 0.03    | 0.04 |
| KQBNKQ  | 1-0 | 1,347         | 5,171      | 0          | 0          | ε                 | ε    | 0       | 0    |
| KQNNKQ  | 1-0 | 49,329        | 38,050     | 0          | 0          | ε                 | 0.01 | 0       | 0    |
|         | 0-1 | 1,538         | 206,733    | 0          | 2          | 0.04              | 0.05 | 0       | ε    |
| KQQRKQ  | 1-0 | 70            | 3,469      | 1,646      | 9,539      | ε                 | ε    | ε       | ε    |
| KQRBKQ  | 1-0 | 153           | 4,061      | 4,771      | 22,119     | ε                 | ε    | ε       | ε    |
| KQRBKR  | 1-0 | 598           | 31,924     | 21,765     | 66,560     | ε                 | ε    | ε       | ε    |
| KQRNKQ  | 1-0 | 654           | 6,196      | 4,707      | 22,857     | ε                 | ε    | ε       | ε    |
| KQRRKQ  | 1-0 | 186           | 4,325      | 2,632      | 14,630     | ε                 | ε    | ε       | ε    |
| KRBBKN  | 1-0 | 237,234       | 45,273,232 | 22,875,477 | 92,309,468 | 0.01              | 1.73 | 1.22    | 3.52 |
| KRBBKQ  | 0-1 | 6,552,902     | 57,721,197 | 434,948    | 2,088,056  | 12.88             | 7.91 | 0.86    | 0.29 |
| KRBBKR  | 1-0 | 4,834         | 29,950     | 115,546    | 131,589    | ε                 | ε    | 0.01    | 0.01 |
| KRBNKN  | 1-0 | 43,735        | 1,208,539  | 2,631,449  | 6,577,857  | ε                 | 0.02 | 0.07    | 0.13 |
| KRBNKQ  | 1-0 | 1,172,828     | 314,964    | 0          | 0          | 0.06              | 0.08 | 0       | 0    |
| KRBNKR  | 1-0 | 6,661         | 30,114     | 190,074    | 226,929    | ε                 | ε    | ε       | 0.01 |
| KRNNKQ  | 0-1 | 33,448        | 252,183    | 10,270     | 30,764     | 0.04              | 0.03 | 0.01    | ε    |
| KRRBKQ  | 1-0 | 102,282       | 248,335    | 0          | 0          | 0.01              | 0.03 | 0       | 0    |
| KRRBKR  | 1-0 | 918           | 30,159     | 76,780     | 179,899    | ε                 | ε    | ε       | 0.01 |
| KRRNKQ  | 1-0 | 225,245       | 274,440    | 0          | 0          | 0.01              | 0.03 | 0       | 0    |
| KRRRKQ  | 1-0 | 1,137         | 4,225      | 0          | 0          | ε                 | ε    | 0       | 0    |

Table 6a. Example Positions showing  $EZ_{50} \neq EZ$ .<sup>18</sup>

| Key                   | Position                        | stm | depth in plies |     |     |                   | Notes                           |
|-----------------------|---------------------------------|-----|----------------|-----|-----|-------------------|---------------------------------|
|                       |                                 |     | dtm            | dtr | dtz | dtz <sub>50</sub> |                                 |
| EZ <sub>50</sub> ≠ EZ |                                 |     |                |     |     |                   |                                 |
| BB-BN                 | 1-0 7b/6nB/8/8/3B4/8/2K5/4k3    | w   | 131            | ?   | 3   | 35                | 1. Bd3?? Ne6" 2. Bxh8 {dtz=52m} |
| BB-NN                 | 1-0 8/8/6n1/8/k3BB2/8/n1K5/8    | w   | 133            | ?   | 1   | 55                | 1. Bxg6?? {dtz=54m}             |
| BN-BN                 | 1-0 5n2/8/8/8/2K2b2/3N4/k3B3    | w   | 11             | 11  | 1   | 11                | 1. Nxf3?? {dtz=70m}             |
| BN-NN                 | 1-0 8/8/8/2B5/2n2N2/2K4n/k7     | w   | 147            | ?   | 1   | 11                | 1. Nxh2?? {dtz=51m} Nd5"        |
| QB-BB                 | 1-0 8/8/5b2/8/8/Q6b/4k2B/K7     | w   | 39             | ?   | 3   | 23                | 1. Be5?? Bxe5+ {dtz=65m}        |
| QB-NN                 | 1-0 8/7Q/8/8/4n3/Bkn5/8/3K4     | w   | 57             | ?   | 7   | 23                | 1. Ke1?? Kxa3 {dtz=52m}         |
| QB-RR                 | 1-0 8/2Kr4/5k2/8/8/5B2/6Q1/3r4  | w   | 213            | 169 | 169 | —                 | a maxDTM/Z pos.                 |
| QN-BB1                | 1-0 8/6bb/5N2/1Q6/5k2/8/8/K7    | w   | 41             | ?   | 3   | 23                | 1. Qb4+?? Kg5 2. Qg4+ Kxf6      |
| QN-BB2                | 0-1 1b6/8/8/K6N/8/8/6Q1/3k1b2   | b   | 129            | ?   | 1   | 7                 | 1. ... Bxg2?? {dtz=52m}         |
| QN-BN                 | 0-1 8/8/8/6Q1/4n3/8/KNk4b       | b   | 5              | 5   | 1   | 5                 | 1. ... Nxg4?? {dtz=53m}         |
| QN-NN                 | 1-0 8/6Q1/4n3/8/2k2n2/3N4/8/2K5 | w   | 37             | ?   | 3   | 15                | 1. Qg4?? Kxd3 {dtz=52m}         |
| QN-RR                 | 1-0 r5r1/8/k7/8/8/3K4/1Q4N1     | b   | 348            | 305 | 305 | —                 | a maxDTM/Z pos.                 |
| QQ-BB                 | 1-0 8/Q7/8/3bb3/8/8/3k4/K4Q2    | w   | 17             | 13  | 3   | 13                | SZ' ×; 1. Qd4+?? Bxd4           |
| QQ-NN                 | 1-0 8/8/8/3n4/Q7/4k3/2K3Q1/4n3  | w   | 69             | ?   | 3   | 7                 | 1. Kd1?? Nxg2 {dtz=52m}         |
| QQ-QB                 | 1-0 7Q/4Q3/8/8/6K1/8/2kq4/5b2   | b   | 142            | 124 | 124 | —                 | a maxDTM/Z pos.                 |
| QQ-QR                 | 0-1 Q2Q4/2K5/8/8/8/r7/1k5q      | b   | 91             | ?   | 1   | 71                | 1. ... Qxa8?? {dtz=60m}         |
| QR-BB                 | 1-0 8/8/5bb1/8/8/Q7/4k3/K2R4    | w   | 35             | ?   | 5   | 19                | 1. Rd4?? Bxd4 {dtz=66m}         |
| QR-NN                 | 1-0 8/8/8/1Q6/3n4/2k5/8/1RK3n1  | w   | 19             | ?   | 1   | 7                 | 1. Rb3+?? Nxb3° {dtz=51m}       |

<sup>18</sup> *stm* ≡ ‘side to move’. Without a DTR EGT, it is not always possible to determine *dtr* precisely.

Table 6b. Example Positions showing  $EZ_{50} \neq EZ$ .

| Key<br>$EZ_{50} \neq EZ$ | Position                        | stm | depth in plies |     |     |                   | Notes                           |
|--------------------------|---------------------------------|-----|----------------|-----|-----|-------------------|---------------------------------|
|                          |                                 |     | dtm            | dtr | dtz | dtz <sub>50</sub> |                                 |
| QR-QB                    | 1-0 8/1Q6/4q3/8/8/6k1/8/1RK4b   | w   | 115            | ?   | 1   | 89                | 1. Qxh1?? {dtz=58m}             |
| QR-QN                    | 1-0 1Q6/8/8/5q2/8/4k3/8/1RK4n   | w   | 101            | ?   | 17  | 75                | 1. Qb6+??                       |
| QR-QR                    | 1-0 8/7R/8/3q4/8/8/1K3k2/Q6r    | w   | 85             | ?   | 1   | 41                | 1. Qxh1?? {dtz=57m}             |
| QR-RB                    | 0-1 8/4R3/5b2/6Q1/8/2k5/6r1/K7  | b   | 7              | 7   | 1   | 7                 | 1. ... Rxg5?? {dtz=56m}         |
| RB-BB                    | 1-0 7k/4R2B/8/8/8/3K2bb/8/8     | w   | 183            | 149 | 149 | —                 | a maxDTM pos.                   |
| RB-BN                    | 1-0 Bb6/8/8/8/8/1R6/3kn3/K7     | b   | 224            | 196 | 196 | —                 | a maxDTM/Z pos.                 |
| RB-NN                    | 1-0 8/8/8/8/2n2k2/2n5/5BR1/1K6  | w   | 475            | 445 | 445 | —                 | a maxDTM/Z pos.                 |
| RB-RB                    | 1-0 1R6/8/8/1b6/8/B7/k1K5/r7    | w   | 23             | ?   | 1   | 15                | 1. Rxb5?? {dtz=54m}             |
| RB-RN                    | 1-0 8/8/3n4/4B3/3K2r1/8/5R2/k7  | w   | 95             | ?   | 9   | 25                | 1. Kc3?? Kb1" 2. Bxd6 {dtz=52m} |
| RN-BB1                   | 1-0 2k1b3/7R/8/8/4NK2/8/8/6b1   | w   | 137            | 103 | 103 | —                 |                                 |
| RN-BB2                   | 0-1 8/3b4/8/8/5b2/K6R/8/1k5N    | b   | 51             | ?   | 1   | 13                | 1. ... Bxh3?? {dtz=53m}         |
| RN-BN1                   | 1-0 NbR5/8/n7/8/8/8/8/2K2k2     | w   | 417            | 379 | 379 | —                 | a maxDTM/Z pos.                 |
| RN-BN2                   | 0-1 2N5/k2R2/8/7b/8/2k5/8/1K2n3 | b   | 163            | ?   | 1   | 11                | 1. ... Bxf7?? {dtz=52m}         |
| RN-NN                    | 1-0 6k1/5n2/8/8/8/5n2/1RK5/1N6  | w   | 523            | 485 | 485 | —                 | a maxDTM/Z pos.                 |
| RR-RB1                   | 1-0 3R4/8/R7/8/8/8/6r1/k3K2b    | b   | 122            | 102 | 102 | —                 |                                 |
| RR-RB2                   | 0-1 8/8/8/1r6/R4b2/6R1/2k5/K7   | b   | 67             | 7   | 1   | 7                 | 1. ... Bxg3?? {dtz=55m}         |
| RR-RN                    | 1-0 2K5/k2RR3/8/8/6n1/8/8/r7    | b   | 178            | 146 | 146 | —                 | a maxDTM/Z pos.                 |
| BBB-N                    | 1-0 8/1B6/8/8/4n3/2Bk3/8/1K6    | w   | 43             | ?   | 2   | 23                | 1. Ba6+?? Kxe3 {dtz=59m}        |
| BBB-Q                    | 0-1 5q2/7K/8/6B1/8/B6B/8/k7     | b   | 91             | ?   | 1   | 59                | 1. ... Qxa3?? {dtz=64m}         |
| BBB-R                    | 1-0 6B1/8/8/6r1/8/7k/7B/K5B1    | w   | 149            | 137 | 137 | —                 | a maxDTM/Z pos.                 |
| BBN-N                    | 1-0 8/8/8/8/5n2/2K5/1N6/1Bk4    | w   | 79             | ?   | 4   | 41                | 1. Bf3 Kxb1 {dtz=55m}           |
| BBN-Q                    | 0-1 8/4K3/7q/1B6/8/3k4/N7/4B3   | b   | 133            | 85  | 13  | 85                | 1. ... Kd4??                    |
| BBN-R                    | 1-0 N7/6B1/8/8/8/7B/1r1k4/K7    | b   | 170            | 136 | 136 | —                 | a maxDTM/Z pos.                 |
| BNN-N                    | 1-0 8/8/8/8/1k6/N7/2K5/N3n2B    | w   | 77             | ?   | 3   | 47                | 1. Kd2?? Kxa3 {dtz=55m}         |
| BNN-Q                    | 0-1 7N/6q1/8/8/2N5/3K1k2/8/B7   | b   | 125            | ?   | 1   | 71                | S(M/Z)σ ×; 1. ... Qxa1??        |
| NNN-B                    | 1-0 6bN/8/8/8/8/1N6/2k5/K6N     | w   | 191            | 183 | 183 | —                 | a maxDTM/Z pos.                 |
| NNN-N                    | 1-0 7N/N7/8/1k6/8/8/2K1n3/1N6   | b   | 180            | 172 | 172 | —                 | a maxDTM/Z pos.                 |
| NNN-Q                    | 0-1 N7/8/8/8/q7/5KN1/8/3k3N     | b   | 127            | ?   | 1   | 41                | 1. ... Qxa8?? {dtz=57m}         |
| QBB-N                    | 1-0 1Q6/8/8/8/7k/BB6/K3n3       | w   | 11             | ?   | 2   | 8                 | 1. Qg3+ ?? Kxg3° {dtz=54m}      |
| QBB-Q1                   | 1-0 8/7K/8/8/2B5/8/1k2Bq2/7Q    | b   | 192            | 186 | 186 | —                 | a maxDTM/Z pos.                 |
| QBB-Q2                   | 0-1 8/Q7/8/8/2B4B/2K5/q7/2k5    | b   | 61             | ?   | 1   | 7                 | 1. ... Qxa7?? {dtz=52m}         |
| QBN-N                    | 1-0 Q7/1B6/8/8/2n5/8/5N2/1k1K4  | w   | 9              | ?   | 2   | 7                 | 1. Qa1+?? Kxa1° {dtz=51m}       |
| QBN-Q                    | 1-0 8/8/2K5/8/8/1Q1B4/8/2kn2q1  | b   | 168            | 130 | 130 | —                 | a maxDTM pos.                   |
| QNN-Q1                   | 1-0 7q/1Q6/8/5N2/8/8/8/K1k4N    | w   | 107            | 101 | 101 | —                 | 1. Ng7" ...                     |
| QNN-Q2                   | 0-1 8/2N5/8/2q5/5N2/2k5/8/2K4Q  | b   | 9              | 7   | 5   | 7                 | SZσ ×; SM ok. 1. ... Qa3+??     |
| QQR-Q                    | 1-0 8/7R/8/8/5q2/7Q/5k2/2K4Q    | w   | 25             | ?   | 2   | 19                | 1. Qe3+ Qxe3+ {dtz=56m}         |
| QRB-Q                    | 1-0 1R5Q/1B6/6k1/5q2/8/8/8/1K6  | w   | 41             | ?   | 3   | 35                | 1. Be4?? Qxe4+ {dtz=51m}        |
| QRB-R                    | 1-0 6B1/8/3r4/8/8/8/3KRQ2/7k    | w   | 63             | ?   | 3   | 18                | 1. Qd4?? Rxd4 {dtz=52m}         |
| QRN-Q                    | 1-0 8/7q/8/8/7N/6k1/2K5/1R5Q    | w   | 83             | ?   | 3   | 67                | 1. Nxf5+?? Qxf5+ {dtz=54m}      |
| QRR-Q                    | 1-0 2R5/3q4/8/8/8/1k6/8/Q2K2R1  | w   | 39             | ?   | 6   | 29                | 1. Kc1?? Qxc8+ {dtz=54m}        |
| RBB-N                    | 1-0 8/8/8/1k6/2R5/1nB5/3K4/7B   | w   | 19             | ?   | 3   | 13                | 1. Kc2?? Kxc4 {dtz=55m}         |
| RBB-Q                    | 0-1 8/8/q7/5K2/8/1B6/3k1B2/2R5  | b   | 131            | ?   | 1   | 29                | 1. ... kxc1 {dtz=55m}           |
| RBB-R                    | 1-0 8/8/8/B7/3K4/8/4R3/2Bk2r1   | w   | 51             | ?   | 7   | 47                | 1. Kd3?? Kxc1 {dtz=55m}         |
| RBN-N                    | 1-0 8/8/8/8/2n4B/8/2N3k1/3K3R   | w   | 25             | ?   | 2   | 13                | 1. Ne1+?? Kxh1° {dtz=76m}       |
| RBN-Q                    | 1-0 1k4q1/8/3K4/8/1N6/8/8/R3B3  | w   | 241            | 197 | 197 | —                 | a maxDTM/Z pos.                 |
| RBN-R                    | 1-0 8/8/8/3R4/1B4r1/1k1K4/N7/8  | w   | 47             | ?   | 7   | 37                | 1. Bd6?? Kxa2" {dtz=54m}        |
| RNN-Q                    | 0-1 7N/R2q4/8/N7/3k4/8/4K3/8    | b   | 125            | ?   | 23  | 65                | 1. Qg4+??                       |
| RRB-Q                    | 1-0 1RK5/1R6/8/1q6/k7/8/7B/8    | b   | 180            | 164 | 164 | —                 | a maxDTM/Z pos.                 |
| RRB-R                    | 1-0 8/8/R7/8/6r1/B7/R2K4/1k6    | w   | 13             | ?   | 2   | 12                | 1. Ra1+?? Kxa1° {dtz=55m}       |
| RRN-Q                    | 1-0 2K5/7k/8/8/4q3/7R/8/5R1N    | b   | 216            | 202 | 202 | —                 | a maxDTM/Z pos.                 |
| RRR-Q                    | 1-0 1R4R1/8/1q6/7R/8/8/5k2/3K4  | b   | 138            | 130 | 130 | —                 | a maxDTM/Z pos.                 |

The following lines, starting from selected positions listed in Table 6, show strategy  $SZ_{50}^-$  delivering the available win while other strategies fail to retain it. They and others were discovered using the Tamplin (2004) web service, and include an established notation showing the criticality of the moves:

" = unique value-preserving move; ' = strategy's only optimal move; ° = only legal move.

Some themes emerge. The attacker can avoid making an ill-advised sacrifice<sup>19</sup> and we include only QRN-Q here. More interestingly, White can delay a capture<sup>20</sup> or go directly for mate<sup>21</sup>. The defender often avoids capturing where, against a fallible player, it would be in its interests to do so to maximize DTR.

KBBKBN position BB-BN –  $dtm = 66m$ ,  $dtz = 2m$ ,  $dtz_{50} = 18m$ :

$S\sigma\tau - SZ_{50}^+$ ,  $\sigma = C$ ,  $M$  or  $Z$ : 1. **Bd3'??** Ne6° 2. Bxh8" { $dtz = 52m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^- - SZ_{50}^+$ : 1. Kd3" Kf1' 2. Bg8" Kg2' 3. Ke4" Kg3' 4. Ba2" Kg4 5. Bb1" Kg3' 6. Bc2' Kg2' 7. Bd1' Kg3 8. Be5+' Kg2' 9. Bg4' Kf2' 10. Kd3' Kf1' 11. Kd2 Kf2' 12. Bd1 Kg2' 13. Ke2 Kh3' 14. Kf2' Kh4' 15. Bf6+' Kh3° 16. Bf3' Kh2' 17. Bg2' Nh5 18. Bxh8" { $dtm = 19m$ } 1-0.

KBBKNN position BB-NN –  $dtm = 67m$ ,  $dtz = 1m$ ,  $dtz_{50} = 28m$ :

$S\sigma\tau - S\phi$ ,  $\sigma = C$ ,  $M$  or  $Z$ : 1. **Bxg6'??** { $dtz = 54m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^- - SZ_{50}^+$ : 1. Bd6" Nh8' 2. Bc6+" Ka5° 3. Kb3" Nc1+' 4. Kc4" Nf7' 5. Bc7+" Ka6° 6. Bd5" Nh8' 7. Bf3' Ng6' 8. Bd6" Nh4' 9. Be4" Ne2' 10. Bh2" Ka5' 11. Bc7+' Ka6' 12. Kc5' Ka7' 13. Bd3' Ng1' 14. Bg3 Ng2' 15. Kc6' Nh3' 16. Bf1' Nh4' 17. Bf2+" Kb8' 18. Bb6' Ka8' 19. Ba6' Kb8' 20. Bc4' Nh5' 21. Bc7+' Ka7 22. Be5' Nh4' 23. Bd6' Nh5 24. Kc7' Nf6' 25. Bc5+' Ka8° 26. Bb5 Nd5+' 27. Kc8" Ne1 28. Bc6#.

KBNKBN position BN-BN –  $dtm = 6m$ ,  $dtz = 1m$ ,  $dtz_{50} = 6m$ :

$SZ^- - SZ_{50}^+$ : 1. **Nxf3'??** { $dtz = 70m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^- - SZ_{50}^+$ : 1. Kb3" Ne6' 2. Bf2" Bd1+ 3. Ka3" Bc2 4. Bb6 Bd1 5. Ba5' Nd4 6. Bc3# 1-0.

KBNKNN position BN-NN –  $dtm = 74m$ ,  $dtz = 1m$ ,  $dtz_{50} = 6m$ :

$SZ^- - SZ_{50}^+$ : 1. **Nxh2'??** { $dtz = 51m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^- - SZ_{50}^+$ : 1. Nd4" Nb1' 2. Be6 Na3+' 3. Kc1" Nf1' 4. Nb3+" Ka2° 5. Nd2+' Ka1' 6. Nxf1' { $dtz = 38m$ ,  $dtm = 68m$ } 1-0.

KQNKBB position QN-BB2 –  $dtm = 65m$ ,  $dtz = 1m$ ,  $dtz_{50} = 4m$ :

$S\phi - S\sigma\tau$ ,  $\sigma = C$ ,  $M$  or  $Z$ : 1. ... **Bxg2??** { $dtz = 52m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^- - SZ_{50}^+$ : 1. ... Bc7+" 2. Kb4' Bd6+" 3. Kc3' Be5+" 4. Kb4 Bxg2' { $dtm = 18m$ } 0-1.

KQNKBN position QN-BN –  $dtm = 3m$ ,  $dtz = 1m$ ,  $dtz_{50} = 3m$ :

$S\phi - SZ\sigma$ : 1. ... **Nxg4'??** { $dtz = 53m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^+ - SZ_{50}^-$ : 1. ... Nc2+" 2. Ka2° Bd5+" 3. Qc4° Bxc4#" 0-1.

KQQKQR position QQ-QR –  $dtm = 46m$ ,  $dtz = 1m$ ,  $dtz_{50} = 36m$ :

$S\phi - SZ\sigma$ : 1. ... **Qxa8'??** { $dtz = 60m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^+ - SZ_{50}^-$ : 1. ... Qh2+" 2. Kd7' Qh3+" 3. Kc7' Qg3+' 4. Kb6' Qe3+' 5. Kb5' Qb3+" 6. Kc5' Qc3+" 7. Kd6' Qd4+" 8. Ke6' Re2+" 9. Kf7 Rf2+" 10. Ke6' Qg4+' 11. Kd5' Rd2+" 12. Kc5' Rc2+" 13. Kd6' Qf4+' 14. Ke6' Re2+" 15. Kd7' Qf5+' 16. Kc7' Rc2+" 17. Kb8' Qf4+' 18. Ka7' Ra2+' 19. Kb6' Rb2+' 20. Kc6' Rc2+' 21. Kb7' Qf7+' 22. Kb8' Rb2+' 23. Kc8' Qc4+' 24. Qc7' Qg4+' 25. Qd7' Rc2+' 26. Kd8' Qg5+" 27. Qe7' Rd2+" 28. Ke8' Qg8+" 29. Qf8° Re2+' 30. Kd7' Qe6+" 31. Kc7' Rc2+" 32. Kb8' Qe5+" 33. Ka7' Qa5+' 34. Kb7' Rc7+ 35. Kb8° Qb6+" 36. Qb7° Qxb7# 0-1.

KQRKQB position QR-QB –  $dtm = 58m$ ,  $dtz = 1m$ ,  $dtz_{50} = 45m$ :

$SZ^- - S\phi$ : 1. **Qxh1'??** { $dtz = 58m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

$SZ_{50}^- - SZ_{50}^+$ : 1. Rb3+" Kf4' 2. Qb4+" Be4' 3. Qd2+" Kg4' 4. Qe2+" Kf5' 5. Qf2+" Ke5' 6. Qg3+' Kd5' 7. Qg5+" Kc6' 8. Rc3+" Kd7' 9. Qg7+" Ke8' 10. Qh8+' Kd7' 11. Rc8' Qg6' 12. Qd8+" Ke6° 13. Qb6+" Ke5' 14. Qb8+' Ke6' 15. Re8+' Kf7' 16. Rf8+" Ke6' 17. Qb6+ Ke7' 18. Qd8+' Ke6° 19. Re8+' Kf5' 20. Qd7+' Kf4' 21. Qd2+' Kf3' 22. Qd1+' Kf4' 23. Qf1+' Ke3' 24. Qe1+" Kd4' 25. Qd2+' Kc4' 26. Qe2+' Kd5' 27. Rd8+' Ke6' 28. Qc4+" Kf5' 29. Rf8+ Ke5' 30. Qc3+' Kd5 31. Rd8+" Ke6' 32. Qc8+' Ke5' 33. Qc7+' Kf5' 34. Rf8+' Ke6' 35. Kb2' Qg2+' 36. Ka3" Bc6' 37. Qf4 Kd7' 38. Qf5+' Kc7 39. Qa5+' Kd6' 40. Rf6+' Kd7' 41. Qa7+' Bb7' 42. Rf7+" Kc8' 43. Qc5+' Kb8 44. Rf8+' Bc8° 45. Rxc8+' { $dtm = 2m$ } 1-0.

<sup>19</sup> e.g., positions QB-BB/NN, QN-BB1/NN, QQ-BB/NN, QR-BB/NN, BBB-N, BBN-N, BNN-N, QBB-N, QBN-N, QQR-Q, QRB-Q/R, QRN-Q, QRR-Q, RBB-N/R, RBN-N/R and RRB-R.

<sup>20</sup> e.g., positions BB-BN, BN-NN, QN-BB2, QR-QB/NN/QR, RB-RB/RN, RN-BN2, RR-RB, BBB-Q, BBN-Q, BNN-Q, NNN-Q, QBB-Q and RBB-Q.

<sup>21</sup> e.g., positions BB-NN, BN-BN, QN-BN, QQ-QR, QR-RB, RN-BB2/BN2 and QNN-Q2.

KQRKQN position QR-QN -  $dtm = 51m$ ,  $dtz = 9m$ ,  $dtz_{50} = 38m$ :

**SZ** – **Sφ**: **1. Qb6+??** Ke2 2. Qa6+ Kf3 3. Qc6+ Kg3 4. Qxh1" { $dtz = 59m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. Qb3+ Kf4 2. Qc3 Qg5 3. Qd2+ Kg4 4. Rb4+ Kh5 5. Rf4 Ng3 6. Kd1 Kh6 7. Qd6+ Kh5 8. Qd4 Nf5 9. Qh8+ Kg6 10. Qe8+ Kf6 11. Qc6+ Ke7 12. Qe4+ Kf6 13. Kc2 Qh5 14. Rf2 Qh3 15. Kb2 Kg5 16. Rg2+ Kf6 17. Qc6+ Ke5 18. Qc7+ Kf6 19. Qd8+ Kf7 20. Qg8+ Kf6 21. Rg6+ Ke5 22. Re6+ Kf4 23. Qb8+ Kg5 24. Qd8+ Kf4 25. Qd2+ Kg4 26. Rg6+ Kf3 27. Rg8 Qh7 28. Qg2+ Kf4 29. Rg4+ Ke5 30. Qe4+ Kd6 31. Qd3+ Ke7 32. Re4+ Kf6 33. Qd8+ Kg6 34. Rg4+ Kf7 35. Qd7+ Ne7 36. Rf4+ Kg8 37. Qe8+ Kg7° 38. Qxe7+ { $dtm = 2m$ } 1-0.

KQRKQR position QR-QR –  $dtm = 43m$ ,  $dtz = 1m$ ,  $dtz_{50} = 21m$ :

**SZ** – **Sφ**: **1. Qxh1??** { $dtz = 57m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>-</sup> – **SZ**<sub>50</sub><sup>+</sup>: 1. Qa7+ Kf3 2. Qa3+ Kg4 3. Qb4+ Kg5 4. Qe7+ Kg4 5. Qg7+ Kf3 6. Qf8+ Ke2 7. Qe8+ Kd3 8. Qg6+ Kc4 9. Qg4+ Kb5 10. Qe2+ Kc5 11. Qe3+ Kc6 12. Qe8+ Kc5 13. Rc7+ Kd4 14. Rd7 Rh2+ 15. Kc1 Rh1+ 16. Kd2 Rh2+ 17. Ke1 Rh1+ 18. Kf2 Rh2+ 19. Kg3 Rg2+ 20. Kh3 Rg5 21. Rxd5+ { $dtz = 29m$ } 1-0.

KQRKRB position QR-RB -  $dtm = 4m$ ,  $dtz = 1m$ ,  $dtz_{50} = 4m$ :

**Sφ** – **SZσ**: **1. ... Rxb5??** { $dtz = 56m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. ... Kb3+ 2. Kbl Rh2+ 3. Kal Ra2+ 4. Kbl° Ra1#0-1.

KRBRB position RB-RB –  $dtm = 12m$ ,  $dtz = 1m$ ,  $dtz_{50} = 8m$ :

**SZ** – **Sφ**: **1. Rxb5??** { $dtz = 54m$ ; Black can 50m-draw} Rg1 2. Bd6 Rg2+ 3. Kc3 Rg6"  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>-</sup> – **SZ**<sub>50</sub><sup>+</sup>: 1. Ra8 Bd3 2. Kc3 Be4 3. Ra4 Kb1 4. Rb4+ Ka2° 5. Bb2 Bc6 6. Rb6 Rh1 7. Ra6+ Ba4 8. Rxa4+ { $dtm = 1m$ } 1-0.

KRBRN – position RB-RN –  $dtm = 48m$ ,  $dtz = 5m$ ,  $dtz_{50} = 13m$ :

**SZ** – **Sφ**: **1. Kc3??** {Black can 50m-draw} Kbl 2. Rf1+ Ka2° 3. Bxd6 { $dtz = 52m$ }.

**SZ**<sub>50</sub><sup>-</sup> – **SZ**<sub>50</sub><sup>+</sup>: 1. Kd3+ Kbl 2. Rb2+ Kc1 3. Ra2 Rb4 4. Bc3 Rb5 5. Re2 Rd5+ 6. Bd4 Kbl 7. Rb2+ Kc1 8. Ra2 Rb5 9. Re2 Kbl 10. Re1+ Ka2° 11. Kc2 Ka3 12. Bc3 Rb2 13. Bxb2+ { $dtm = 15m$ } 1-0.

KRNKBB position RN-BB2 -  $dtm = 26m$ ,  $dtz = 1m$ ,  $dtz_{50} = 7m$ :

**Sφ** – **SZσ**: **1. ... Bxb3??** { $dtz = 53m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. ... Kc2 2. Rh5 Bd6+ 3. Ka2 Be6+ 4. Kal Bb4 5. Rh2+ Kc1 6. Rf2 Bc3+ 7. Rb2° Bxb2# 0-1.

KRNKBN position RN-BN2 -  $dtm = 82m$ ,  $dtz = 1m$ ,  $dtz_{50} = 6m$ :

**SZ**<sub>50</sub><sup>+</sup> – **SZ**: **1. ... Bxf7??** { $dtz = 52m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. ... Bg6+ 2. Kal Nc2+ 3. Kbl Nb4+ 4. Kcl Nd3+ 5. Kdl Bh5+ 6. Rf3° Bxf3#.

KRRKRB position RR-RB –  $dtm = 34m$ ,  $dtz = 1m$ ,  $dtz_{50} = 4m$ :

**Sφ** – **SZσ**: **1. ... Bxb3??** { $dtz = 55m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. ... Be5+ 2. Ka2 Rb2+ 3. Ka3 Bd6+ 4. Rb4° Rxb4 { $dtm = 30m$ } 0-1.

KBBBBKQ position BBB-Q –  $dtm = 46m$ ,  $dtz = 1m$ ,  $dtz_{50} = 30m$ :

**Sφ** – **SZσ**: **1. ... Qxa3??** { $dtz = 64m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. ... Qf3 2. Bc8 Qh5+ 3. Bh6 Qf7+ 4. Bg7+ Ka2 5. Baf8 Qd5 6. Kh8 Kb3 7. Bh6 Ka4 8. Bh3 Kb5 9. Kg7 Qe5+ 10. Kh7 Qc7+ 11. Bfg7 Qc2+ 12. Kh8 Qe4 13. Bf8 Kb6 14. Kg7 Qe5+ 15. Kh7 Qc7+ 16. Bfg7 Qc2+ 17. Kh8 Qe4 18. Bf8 Kb7 19. Kg7 Qe5+ 20. Kh7 Qc7+ 21. Bfg7 Qc2+ 22. Kh8 Qg6 23. Bf8 Kc7 24. Bf4+ Kd8 25. B8h6 Ke7 26. Bf1 Qc2 27. Kg7 Qb2+ 28. Kg8 Qa2+ 29. Kg7 Qa1+ 30. Kg6 Qxf1 { $dtm = 17m$ } 0-1.

KBBNKN position BBN-N -  $dtm = 40m$ ,  $dtz = 2m$ ,  $dtz_{50} = 21m$ :

**SZ** – **SZ**<sub>50</sub><sup>+</sup>: **1. Bf3??** Kxb1 { $dtz = 55m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>-</sup> – **SZ**<sub>50</sub><sup>+</sup>: 1. Bh7 Ne2+ 2. Kb3 Nd4+ 3. Ka2 Kd2 4. Ka3 Kc3 5. Na4+ Kd2 6. Bg4 Ke3 7. Kb4 Nc6+ 8. Kc5 Ne5 9. Bh3 Nf7 10. Kd5 Ng5 11. B7f5 Kf4 12. Nc5 Kg3 13. Bhg4 Nf7 14. Bh5 Nh6 15. Ke6 Kf4 16. Bfg6 Ke3 17. Kf6 Kd4 18. Na6 Ng8 19. Kf7 Nh6+ 20. Kg7 Ng4 21. Bxg4 { $dtm = 17m$ } 1-0.

KBBNKQ position BBN-Q –  $dtm = 67m$ ,  $dtz = 7m$ ,  $dtz_{50} = 43m$ :

**SZ**<sub>50</sub><sup>+</sup> – **SZ**: **1. ... Kd4??** 2. Bf2+ Ke5 3. Bg3+ Kd5 4. Nc3+ Kd4 5. Bd6 Kxc3 { $dtz = 51m$ ; White can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>+</sup> – **SZ**<sub>50</sub><sup>-</sup>: 1. ... Ke3 2. Be8 Qg5+ 3. Kf8 Qc5+ 4. Kg8 Qc8 5. Kf8 Qa8 6. Nb4 Qa3 7. Kg7 Qb2+ 8. Kf7 Qb3+ 9. Kg7 Kf4 10. Bd2+ Kg4 11. Bd7+ Kh5 12. Be8+ Kh4 13. Be1+ Kg4 14. Bd7+ Kf3 15. Bf5 Kf4 16. Bh7 Qb2+ 17. Kg6 Kg4 18. Nd3 Qd4 19. Kf7 Qd7+ 20. Kg8 Qe8 21. Kg7° Qe7+ 22. Kg6 Qg5+ 23. Kf7° Kh5 24. Bb4 Qd5+ 25. Kg7 Qd4 26. Kg8 Qg4 27. Kh8 Qc8+ 28. Bg8 Qc7 29. Bd2 Qd6 30. Nf4+ Kg4 31. Bd5 Kf5 32. Kg8 Qb8+ 33. Kf7 Qc7+ 34. Kg8 Qc2 35. Be6+ Ke5 36. Be3 Qe4 37. Bcl Kf6 38. Bb2+ Kg5 39. Bcl Qa8+ 40. Kf7 Qb7+ 41. Kf8 Kf6 42. Ba3 Qa8+ 43. Bc8° Qxc8# 0-1.

KBNNKN position BNN-N -  $dtm = 39m$ ,  $dtz = 2m$ ,  $dtz_{50} = 24m$ :

**SZ** – **SZ**<sub>50</sub><sup>+</sup>: **1. Kd2??** Kxa3 { $dtz = 55m$ ; Black can 50m-draw}  $\frac{1}{2}-\frac{1}{2}$ .

**SZ**<sub>50</sub><sup>-</sup> – **SZ**<sub>50</sub><sup>+</sup>: 1. Kb2 Nd3+ 2. Ka2 Nc1+ 3. Kbl Nd3 4. Nlc2+ Kc5 5. Ba8 Kd6 6. Ne3 Kc5 7. Kc2 Nb4+ 8. Kc3 Na2+ 9. Kd2 Nb4 10. Nac4 Na6 11. Kd3 Nb4 12. Ke4 Nc6 13. Ne5 Na7 14. Nd3+ Kd6 15. Nc4+ Kc7 16. Nb4 Kb8 17. Bd5 Nb5 18. Bc6 Na7 19. Ba4 Nc8 20. Ke5 Ka7 21. Ke6 Kb8 22. Kd7 Kb7 23. Nd6+ Kb6 24. Nxc8+ { $dtm = 28m$ } 1-0.

KBNNKQ position BNN-Q -  $dtm = 63m$ ,  $dtz = 1m$ ,  $dtz_{50} = 36m$ :

**Sφ - Sσ, σ = C', M' or Z': 1. ... Qxa1'??** { $dtz = 52m$ ; White can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>+</sup> - SZ<sub>50</sub><sup>-</sup>:** 1. ... Qh7+" 2. Kd2' Qd7+" 3. Kc3' Ke2' 4. Bb2' Qg4" 5. Kb3' Qe6" 6. Kc3' Qe4" 7. Kb3' Qg4' 8. Kc3' Qf4' 9. Kb3' Qb8+" 10. Kc2' Qb4' 11. Na3' Qe4+" 12. Kb3' Qd5+" 13. Kc3' Qf3+" 14. Kc4' Kd1 15. Kb4' Qb7+" 16. Nb5' Kc2' 17. Bd4' Qe7+" 18. Kc4' Qe6+" 19. Kc5' Qf5+" 20. Kc4' Qc8+" 21. Kb4' Qf8+" 22. Ka4' Qg8' 23. Kb4' Kd3' 24. Bc3' Qd5' 25. Bd4' Qc4+" 26. Ka5' Qg8' 27. Ka4' Qa8+" 28. Kb4' Qf8+" 29. Kb3' Qe7' 30. Bb2' Qe6+" 31. Ka4' Qa2+ 32. Ba3' Qc4+" 33. Ka5' Qd5' 34. Kb4' Qe4+ 35. Ka5' Qa8+" 36. Kb6' Qxh8 { $dtm = 22m$ } 0-1.

KNNNKQ position NNN-Q -  $dtm = 64m$ ,  $dtz = 2m$ ,  $dtz_{50} = 21m$ :

**Sφ - SZ': 1. ... Qxa8'??** { $dtz = 57m$ ; White can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>+</sup> - SZ<sub>50</sub><sup>-</sup>:** 1. ... Qa3+" 2. Kf4' Qd6+" 3. Kg4' Qd4+" 4. Kf3' Qf6+" 5. Kg4' Qg7+" 6. Kf3' Kd2' 7. Ne4+" Kd3" 8. Nc5+" Kc4' 9. Nd7' Qf7+" 10. Ke3' Qe6+" 11. Kf2' Qf5+" 12. Kg2' Qd5+" 13. Kh2' Qd2+" 14. Kg3' Kd3' 15. Nab6' Ke2" 16. Kg2' Qb2' 17. Kg3' Qd4' 18. Kg2' Qe4+" 19. Kh2' Kf3' 20. Nf8' Qg4 21. Ng3' Qxg3+" { $dtm = 1m$ } 0-1.

KQBBKQ position QBB-Q2 -  $dtm = 31m$ ,  $dtz = 1m$ ,  $dtz_{50} = 4m$ :

**Sφ - SZ': 1. ... Qxa7'??** { $dtz = 52m$ ; White can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>+</sup> - SZ<sub>50</sub><sup>-</sup>:** 1... Qd2+" 2. Kb3° Qb2+" 3. Ka4° Qa1+" 4. Kb3' Qxa7' { $dtm = 27m$ } 0-1.

KQNNKQ position QNN-Q2 -  $dtm = 4m$ ,  $dtz = 3m$ ,  $dtz_{50} = 4m$ :

**SZ<sub>50</sub><sup>+</sup> - SZ': 1. ... Qa3+'??** 2. Kd1" Qa1+" 3. Ke2° Qxh1" { $dtz = 52m$ ; White can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>+</sup> - SZ<sub>50</sub><sup>-</sup>:** 1. ... Qe3+" 2. Kb1' Qb6+" 3. Kc1' Qb2+" 4. Kd1° Qd2#" 0-1.

KQRNKQ position QRN-Q -  $dtm = 42m$ ,  $dtz = 2m$ ,  $dtz_{50} = 34m$ :

**SZ' - SZ<sub>50</sub><sup>+</sup>:** 1. **Nf5+'??** {unnecessary sac.} 1... Qxf5+" { $dtz = 54m$ ; Black can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>-</sup> - SZ<sub>50</sub><sup>+</sup>:** 1. Kb3' Qd3+ 2. Kb4' Qd4+" 3. Kb5' Qd7+" 4. Ka6' Qd6+" 5. Ka5' Qa3+" 6. Kb5' Qd3+ 7. Kc6' Qc4+" 8. Kb7' Qf7+" 9. Kb6' Qf2+" 10. Kc6' Qf6+" 11. Kb5' Qe5+" 12. Ka4' Qd4+" 13. Rb4' Qa7+" 14. Kb3' Qe3+" 15. Ka2' Qe2+" 16. Rb2' Qa6+" 17. Kb1' Qd3+" 18. Rc2' Qb3+" 19. Kc1' Qa3+" 20. Kd2' Qd6+" 21. Kc3' Qc5+" 22. Kd3' Qd6+" 23. Kc4' Qa6+" 24. Kd5' Qb5+" 25. Rc5' Qb3+ 26. Kd6' Qb8+" 27. Kd7' Qa7+" 28. Rc7' Qd4+ 29. Ke6" Qe3+" 30. Kf7' Qb3+" 31. Kg7' Qb2+" 32. Kh7' Qb4 33. Qf3+" Kh2' 34. Qg2#" 1-0.

KRBBKQ position RBB-Q -  $dtm = 66m$ ,  $dtz = 1m$ ,  $dtz_{50} = 15m$ :

**SZ<sub>50</sub><sup>+</sup> - SZ': 1. ... Kxc1'??** { $dtz = 55m$ ; White can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>-</sup> - SZ<sub>50</sub><sup>+</sup>:** 1. ... Qd3+" 2. Kf4' Qd6+" 3. Kf5' Qf8+" 4. Ke4' Qe8+" 5. Kd4' Qh8+" 6. Ke4' Qh7+" 7. Ke5' Qh2+" 8. Kd5' Qg2+" 9. Kc4' Qg8+" 10. Kc5' Qf8+" 11. Kc4' Qf7+" 12. Kb4' Qb7+" 13. Kc4' Qc7+" 14. Kd4' Qf4+" 15. Kd5' Kxc1' { $dtm = 16m$ } 0-1.

KRNNKQ position RNN-Q -  $dtm = 63m$ ,  $dtz = 12m$ ,  $dtz_{50} = 33m$ :

**SZ<sub>50</sub><sup>+</sup> - SZ': 1. ... Qg4+'??** 2. Kd2" Qg2+" 3. Kc1" Qh2" 4. Rf7" Qg3" 5. Kd1' Qd3+" 6. Ke1" Ke3' 7. Re7+" Kf3" 8. Nf7" Qb1+" 9. Kd2° Qb4+" 10. Kd3" Qxe7" { $dtz = 52m$ ; White can 50m-draw}  $\frac{1}{2}$ - $\frac{1}{2}$ .

**SZ<sub>50</sub><sup>+</sup> - SZ<sub>50</sub><sup>-</sup>:** 1. ... Qe8+" 2. Kd2' Qe3+" 3. Kc2' Qc3+" 4. Kb1' Kd3' 5. Rd7+' Ke3" 6. Re7+' Kf2' 7. Rf7+' Kg1' 8. Nb7' Qd2' 9. Rg7+' Kf1' 10. Rc7' Qb4+" 11. Ka2' Qa4+" 12. Kb2' Qd4+" 13. Kc2' Qf2+" 14. Kd1' Qe2+" 15. Kc1° Qe5 16. Rf7+ Ke1 17. Kc2' Qe3 18. Kb2' Qd3 19. Ka2' Qc3 20. Re7+ Kd2 21. Rf7' Qb4 22. Rd7+ Ke2 23. Rc7' Qb6 24. Rc2+ Kd1 25. Rb2' Qa6+ 26. Kb1' Qd3+ 27. Ka2' Qc4+ 28. Ka1' Qa4+ 29. Kb1' Qe4+ 30. Ka2' Kc1 31. Ka3' Qd3+ 32. Rb3' Qa6+ 33. Kb4' Qxb7+ { $dtm = 30m$ } 34. Ka4' Qa8+ 35. Kb4' Qxh8 { $dtc = 24m$ } 0-1.