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Abstract
Recent research has suggested that forecast evaluation on the basis of standard statistical loss functions could prefer models which are sub-optimal when used in a practical setting. This paper explores a number of statistical models for predicting the daily volatility of several key UK financial time series. The out-of-sample forecasting performance of various linear and GARCH-type models of volatility are compared with forecasts derived from a multivariate approach. The forecasts are evaluated using traditional metrics, such as mean squared error, and also by how adequately they perform in a modern risk management setting. We find that the relative accuracies of the various methods are highly sensitive to the measure used to evaluate them. Such results have implications for any econometric time series forecasts which are subsequently employed in financial decision-making.

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I. Introduction

Modelling and forecasting stock market volatility has been the subject of a great deal of debate over the past fifteen years or so. Volatility, usually measured by the standard deviation of portfolio returns, is uniquely important in financial markets, for it is often taken to represent the portfolio's risk. Consequently, the literature on forecasting volatility is sizeable and still growing. Akgiray (1989), for example, finds the GARCH model superior to ARCH, exponentially weighted moving average, and historical mean models for forecasting monthly US stock index volatility. A similar result concerning the apparent superiority of GARCH is observed by West and Cho (1995) using one-step ahead forecasts of Dollar exchange rate volatility, evaluated using root-mean squared prediction errors. However, for longer horizons, the model behaves no better than their alternatives[1]. Also using the same models and data, West *et al*. (1993) use asymmetric, utility-based criteria for evaluating the conditional variance forecasts, finding that GARCH models tend to yield the highest utilities. Pagan and Schwert (1990) compare GARCH, EGARCH, Markov switching regime and three non-parametric models for forecasting monthly US stock return volatilities. The EGARCH followed by the GARCH models perform moderately; the remaining models produce very poor predictions. Franses and van Dijk (1996) compare three members of the GARCH family (standard GARCH, QGARCH and the GJR model) for forecasting the weekly volatility of various European stock market indices. They find that the non-linear GARCH models were unable to beat the standard GARCH model. Brailsford and Faff (1996) find GJR and GARCH models slightly superior to various simpler models[2] for predicting Australian monthly stock index volatility. The conclusion arising from this growing body of research is that forecasting volatility is a "notoriously difficult task" (Brailsford and Faff, 1996, p419), although it appears that conditional heteroscedasticity models are among the best that are currently available. In particular, more complex non-linear and non-parametric models are inferior in prediction to simpler models, a result echoed in an earlier paper by Dimson and Marsh (1990) in the context of relatively complex versus parsimonious linear models. Finally Brooks (1998) uses a measure of market volume in volatility forecasting models, but observes no increase in forecasting power.

[1] The alternative models are the long term mean, IGARCH, autoregressive models, and a nonparameteric model based on the Gaussian kernel.
[2] The other models employed are the random walk, the historical mean, a short- and a long-term moving average, exponential smoothing, an exponentially weighted moving average model, and a linear regression.

Recent papers have also sought to compare the predictive ability of volatility forecasts derived from the market prices of traded options, with those generated using econometric models (see, for example, Heynen and Kat, 1994 or Day and Lewis, 1992). The general consensus appears to be that implied volatility forecasts are more accurate from those derived using pure time series analysis, but also that the latter still contain additional information not embedded in the implied values.

Also over the past decade, there has been rapid development of techniques for measuring and managing financial risk, partially motivated by a spate of recent financial disasters involving derivative securities. One of the most popular approaches to risk measurement is by calculating what is known as an institution's "value at risk" (VaR). Broadly speaking, value at risk is an estimation of the probability of likely losses which could arise from changes in market prices. More precisely, it is defined as the money-loss in a portfolio that is expected to occur over a pre-determined horizon and with a pre-determined degree of confidence. The roots of VaR's popularity stem from the simplicity of its calculation, its ease of interpretation, and from the fact that VaR can be suitably aggregated across an entire firm to produce a single number which broadly encompasses the risk of the positions of the firm as a whole. Jorion (1996) or Dowd (1998) provide thorough introductions to value at risk, and Brooks and Persand (2000a and 2000b) present recent discussions of VaR model estimation issues. The value at risk estimate is also often known as the position risk requirement or minimum capital risk requirement (MCRR); we shall use the three terms interchangeably in the exposition below.

Although the academic literature has thus far failed to keep pace with this expansion, evidenced by the relatively few academic studies that address this topic, one exception is the study by Jackson *et al*. (1998), which assesses the empirical performance of various models for value at risk using historical returns from the actual portfolio of a large investment bank. They find that non-parametric, simulation-based techniques yield more accurate measures of the tail probabilities than parametric models. Alexander and Leigh (1997) offer an analysis of the relative performance of equally weighted, exponentially weighted moving average (EWMA), and GARCH model forecasts of volatility, evaluated using traditional statistical and operational adequacy criteria. The GARCH model is found to be preferable to EWMA in terms of minimising the number of exceedences in a backtest, although the simple unweighted average is superior to both. Brooks, Henry and Persand (2001) investigate the effectiveness of various hedging models when assessed according to their ability to minimise VaR, finding that there is a large role for time-varying volatilities and correlations, but a very minor role for asymmetries.

This paper seeks to combine and advance the two literatures in volatility forecasting and financial risk management in a number of ways. First, the volatility forecasting debate is re-opened, and the forecasts from the various models evaluated on the basis of how well they perform in a modern risk management setting, as well as by traditional statistical loss functions. This is important for Dacco and Satchell (1999) demonstrate that the evaluation of forecasts from nonlinear models using statistical measures can be misleading, and they propose the use of alternative economic loss functions. Here, the relative performances of the forecasting models are evaluated using both statistical and economic loss functions, so that a comparison can be drawn between the two. Second, we also directly compare the forecasting performance of univariate and multivariate forecasting models for financial asset return volatility. Multivariate GARCH models permit the estimation of the conditional covariances between assets' returns, and explicit modelling of this interaction may improve the accuracy of forecasts of volatility for a portfolio comprising these components. Finally, we evaluate forecasts over the 1- 5-, 10- and 20-day horizons. Although many volatility forecasting papers compare accuracies at daily horizons, it is often the case that financial market practitioners require predictions of much lower frequency. For example, the Basle Committee on Banking Supervision rules for the use of VaR models (see, for example Basle Committee on Banking Supervision, 1998) require the use of a 10-day holding period, which allows reasonable time for investors to unwind a position, and fund managers typically re-balance their portfolios on a monthly (20 trading days) basis.

The remainder of the paper is organised as follows. Section 2 presents the data employed in the study, while the forecasting models are described briefly in section 3. Forecast evaluation methods are outlined and discussed in section 4, with results given in section 5. Finally, section 6 summarises the paper, and offers some concluding remarks.

2. The Data

In this study we calculate the VaRs for three different assets - the FTSE All Share Total Return Index, the FTA British Government Bond (over 15 years) Index and the Reuters Commodities Price Index, as well as an equally weighted portfolio containing these three assets[3, 4]. The data was collected from Datastream International, and spans the period 1st January 1980 to 25th March 1999. Observations corresponding to UK public holidays were deleted from the data set to avoid the incorporation of spurious zero returns, leaving 4865 observations, or trading days in the sample. In the empirical work below, we use the daily log return of the original indices. Summary statistics for the data are given in table 1. It is evident that the FTSE returns series is the most volatile, while the government bond index returns is the least. The benefits from diversification, in terms of a substantial reduction in variability, are clear, since the variance for the equally weighted portfolio returns is almost half that of the least volatile component. Also, as one might anticipate, the series are all strongly non-normal. All are leptokurtic, while the FTSE All-Share and commodities series are also significantly skewed to the left.

3. Forecasting Volatility

3.1 Construction of Forecasts and Notation

The total sample of 4865 observations is split into two parts: the first 1250 observations (approximately 5 years of daily trading data) are used for estimation of the parameters of the model, and then one, two,…, twenty step ahead forecasts are calculated. The multi-step ahead forecasts are then aggregated to form forecasts of volatility over the next 5, 10, and 20 days. We can thus write[5]

$$\sigma_{t,N}^2 = \sum_{n=1}^{N} \sigma_{t,t+n}^2 \qquad (1)$$

where $\sigma_{t,N}^2$ denotes the time t aggregated forecast for the next N steps, and $\sigma_{t,t+n}^2$ denotes the n step ahead forecast made at time t.

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[3] Our analysis assumes that we are long all the three assets - both individually and in the portfolio. A similar analysis could be undertaken for short or netted positions, but we would not expect our conclusions to be markedly altered.

[4] This portfolio is deliberately highly simplistic relative to a genuine bank's book, as well as being entirely linear in nature. The use of a simple portfolio enables us to more easily unravel the various estimation issues and broad aspects of the methodologies. Additionally, the three series that we consider are all fundamental or "benchmark" factor series, to which other series are mapped under the JP Morgan approach.

[5] This step is permissible since the variances are additive over time. Another possibility would be to multiply the one step ahead forecast by the desired horizon using an equivalent of the "square root of time" rule, so that, for example, the volatility forecast over the next 20 days is 20 times the forecast for tomorrow. However, our approach is likely to be superior, since it employs more information while implicit extrapolation of one step forecasts could be inappropriate for a mean-reverting series.

In contrast to much previous research in this area, these are not 1,2,3,…,20 step ahead forecasts, but rather we aggregate the forecasts for the next 5, 10, and 20 days. Aggregated forecasts will be the ones of interest to financial market practitioners and risk managers, when they have investment horizons longer than one day; they will not be particularly interested in multi-step ahead one-day volatility forecasts, such as the volatility forecast for day t+20 made on day t. The sample is then rolled forward by removing the first observation of the sample and adding one to the end, and another set of forecasts of the next twenty days' volatilities is made, and aggregated. This "recursive" modelling and forecasting procedure is repeated until a forecast for observation 4865 has been made using data available at time 4845. Computation of forecasts using a rolling window of data should ensure that the forecasts are made using models whose parameters have been estimated using a sufficient span of time, while not incorporating such old vintages that the data may no longer relevant in the context of an evolving financial market.

3.2. Forecasting Models

Almost all of the forecasting models employed in this study are not new, rather it is the evaluation of the models which is novel. Hence the model descriptions are brief and presented in table 2, with $\sigma^2_{f,t+n}|\Omega_t$ denoting the n-step ahead (n = 1, 2, …, 20) forecast for the conditional variance upon information available at time t, where t runs from observation 1250 to 4845. With one possible exception, the model equations in table 2 are self-explanatory, and readers are referred to Bollerslev *et al*. (1992), Brailsford and Faff (1996), or Brooks (1998), and the references therein, for a more detailed treatment.

The only model which perhaps requires further explanation is the multivariate GARCH model, which has not been employed in previous studies of volatility forecast performance. The particular parameterisation used here is of the diagonal VEC form due to Bollerslev, Engle and Wooldridge (1988), where each element of the conditional variance covariance matrix $h_{jk,t}$ depends on past values of itself and past values of $\varepsilon_{j,t}\varepsilon_{j,t}$' , which may be written

$$vec(H_{t+1}) = h_{t+1} = C_0 + A_1 vec(\varepsilon_t \varepsilon_t') + B_1 h_t \quad (14)$$

where vec denotes the column stacking operator, A_1 and B_1 are restricted to be diagonal. The parameterisation for H_{t+1} conditional upon the information set, allows each element of the conditional variance-covariance matrix to depend on lags of the squares and of the cross products of the elements of ε_{t+1} as well as lags of the elements of H_{t+1}.

4. Evaluating Volatility Forecasts

4.1 Standard Loss Functions

Three criteria are used here to evaluate the accuracy of the forecasts: mean squared error (MSE), mean absolute error (MAE), and proportion of over-predictions. Mean squared error provides a quadratic loss function which disproportionately weights large forecast errors more heavily relative to mean absolute error, and hence the former may be particularly useful in forecasting situations when large forecast errors are disproportionately more serious than small errors. The proportion of over-predictions should give a rough indication of the average direction of the forecast error (compared with the two previous measures which only give some measure of the average size) and whether the models are persistently over- or under-predicting the "true" value of volatility. Hence this measure gives an approximate guide as to whether the forecasts are biased.

4.2 But What is Volatility?

Unlike financial asset returns, volatilities are not directly observable from the market. Consequently, when attempting to benchmark the accuracy of volatility forecasting models, researchers are necessarily required to make an auxiliary assumption about how the *ex post*, or realised volatilities are calculated. The vast majority of existing studies, including those listed in the introduction to this paper, use squared returns of the frequency of the data and analysis, as the measure of realised volatility. For example, studies using daily data would assume that the "correct" volatility number on day *t* is r_t^2, and it is this value that would be used as an input to the mean squared error calculation, or as the dependent variable in a Fair-Schiller (1990)-type regression of actual volatilities on their forecasted values.

Whilst this method is simple and intuitively plausible, Andersen and Bollerslev (1998, hereafter AB) suggest that "same-frequency" squared returns are an unbiased but extremely noisy measure of the latent volatility factor which underlies financial asset return movements. AB show that a much better approximation to the latent volatility factor can be obtained by summing the squares of higher frequency returns. For example, a superior estimate of volatility on day *t* to r_t^2 is given by

$$r_t^{\ 2}* = \sum_{j=1}^{m} r_{t-1+(j/m)}^2 \qquad (15)$$

where m is an intra-day sampling frequency, such as 8 for hourly data[6].

Unfortunately, for many applications, the usefulness of this method is limited by the lack of availability of a sufficiently long span of higher-frequency returns. In the present paper, however, our analysis focuses upon daily, weekly, bi-weekly, and monthly forecasts. For the latter three horizons, two methods of calculating *ex post* volatility are available, both of which are employed in this study. The first of these *ex post* measures, which may usefully be termed the traditional measure, is to use weekly, bi-weekly or monthly squared returns[7]. The second method, would be to take the daily returns, square them, and sum them over the relevant (5, 10, or 20 day) horizon[8]. As AB note, it is not necessarily the case that the two *ex post* measures will give the same model rankings, let alone the same values of the error measures. Thus a comparison of model rankings under the two approaches is a relevant question for research, which this paper makes the first attempt to address.

4.3 Value at Risk Calculation

Given the voluminous literature which almost unquestioningly evaluates volatility forecasts using standard loss functions, three sensible questions to ask are first, what are volatility forecasts useful for, second, what is an appropriate loss function given this usage, and finally, will alternative loss functions lead to approval of the same or similar models? Some answers to the first of these questions are provided in the introduction to this paper. One use of volatility predictions, which has grown substantially in importance over recent years, is as an input to financial risk management. In this paper, we thus employ a relevant "risk management" loss function, which is based upon the calculation of an institution's value at risk, as defined above in section 1. Specifically, we calculate value at risk for three individual assets by calculating the following quantity

$$VAR_t^i(N,\alpha) = [F_{t,N}^i]^{-1}\left(\frac{\alpha}{100}\right) \qquad (16)$$

[6] Assuming, of course, that 8 hourly observations are available from the financial market concerned.
[7] So, for example, the volatility for weekly returns would be given by $r_t^2 = [\ln(P_t / P_{t-5})]^2$.
[8] Obviously for the 1-day horizon, both methods will yield the same *ex post* measure.

where VAR_t^i is the value at risk for a given asset at time *t*, determined from model *i* (where i = 1, 13 are the models as defined in section 3.2 above), *N* is the investment horizon, $[F_{t,N}^i]^{-1}$ is a cumulative distribution function (cdf) and α is a percentage significance level. The cdf employed in this paper is that of a normal distribution.

A limiting assumption of the analysis in many empirical papers in risk management is the standard assumption of normality, for it is well known that asset returns are not Gaussian. However, the normal approximation is extremely widely used in the risk management field.Fat tailed return distributions will lead the delta-normal model to understate the true value at risk (see, Jorion, 1996 or Huisman *et al*., 1998). For Example, a 5% daily loss is observed to occur approximately once every two years, while if returns were normally distributed, such a change would be expected only once every 1000 years (Johansen and Sornette, 1999). A number of methods to incorporate the fat tails have been proposed, most importantly the use of extreme value distributions for returns (e.g. Embrechts *et al*., 1999). However, we continue to employ the normality assumption since other distributional approaches usually do not directly employ a volatility estimate. Therefore our purpose of comparing between volatility forecasts when used for risk management would be lost.

We employ both the 1% and 5% levels of significance. The former level has been selected by the Basle Committee (1996) as the focus of attention, although the first percentile of a distribution is more difficult to estimate than the fifth, and thus the latter is the quantity which many securities firms wish to employ (see JP Morgan, 1996). The VaR corresponding to 5% may be defined as that amount of capital, expressed as a percentage of the initial value of the position, which will be required to cover 95% of probable losses. In the case of the normal distribution, this quantity may be calculated as

$$VAR_t^i(N,5\%) = 1.645\sigma_{t,N}^i \tag{17}$$

where $\sigma_{t,N}^i$ is the square root of the conditional variance forecast, made at time *t* for forecast horizon *N* (*N*=1,5,10,20). We thus forecast volatility for some future period (*t*,*N*) and hence we calculate the amount of capital required to cover expected losses on 95% or 99% of the investment horizons. The 95% confidence level is employed by the popular RiskMetrics^{TM} risk measurement software, while the regulators require capital to cover 99% of losses[9].

The calculation of the value at risk estimates for the individual assets is achieved by following the steps outlined above. In the case of the portfolio, however, for all forecasting models except the mutlivatiate GARCH (that is models 1-12 in Table 2), we employ a method known as the "full valuation approach". This simply involves the aggregation of the components and the calculation of the portfolio return at each point in time. In this case, the resulting portfolio return series is modelled in the same way as the individual component assets.

An alternative approach is known as the "volatilities and correlations" method, which has been popularised by J.P. Morgan (1996). Here, the portfolio value at risk is estimated using the volaitlities of the individual assets which form the MCRR, and the correlations between their returns. The portfolio value at risk may be written

$$MCRR_P = \sqrt{\begin{array}{l} a^2 MCRR_A^2 + b^2 MCRR_B^2 + c^2 MCRR_C^2 \\ + 2ab\rho_{AB} MCRR_A MCRR_B \\ + 2ac\rho_{AC} MCRR_A MCRR_C \\ + 2bc\rho_{BC} MCRR_B MCRR_C \end{array}} \quad (18)$$

where *A*, *B*, and *C* denote the bond, stock and commodities series respectively, and $a=b=c=1/3$. We adopt this approach when using the multivariate GARCH model, but instead of using the time-invariant volatility and correlation estimates, we instead use the relevant forecasts of the conditional variances and covariances from the MGARCH model in (18) to derive the VaR.

4.4 Risk Management-Based Forecast Evaluations

In this paper, we employ three methods for determining the adequacy of the volatility forecasts that are used as an input to the value at risk calculation. All methods essentially require the calculation of VaR, and then assuming that the securities firm had employed this much capital, the methods track the actual realised losses

[9] In fact, the 99% VaR is multiplied by a "scaling factor" , which is usually 3, so that the actual coverage rate is considerably higher than 99%. We do not employ the regulatory scaling factor in our analysis, so as to focus upon forecast adequacy. Multiplying the estimated VaR by 3 has the effect of rendering the forecasted VaRs virtually indistinguishable from one another, since the implied coverage rate is now more than 99.99%.

during an out of sample period. The simplest approach to determining model adequacy in the risk management framework is to calculate the time until first failure (TUFF), defined as the first observation in the hold out sample where the capital held is insufficient to absorb that period's loss, and derived as follows. Following Kupiec (1995), let p denote the realised probability of observing the first failure of the model in period V, and letting $\tilde{R}$ be a random variable that denotes the number of observations until the first failure is recorded, then we may write

$$\Pr(\tilde{R} = V) = p(1-p)^{V-1} \tag{19}$$

Then $\tilde{R}$ has a geometric distribution with an expected value of $1/p$. This quantity can be interpreted as the expected number of observations until the first failure is observed. In the cases of interest in this paper, if the actual proportion of failures were 5% and 1% respectively, then the time until first failure would be 100 and 20 steps respectively. If we now let p^* denote the probability of failure under the null hypothesis, then the following likelihood ratio test can be established

$$TUFF(V, p^*) = -2\log[p^*(1-p^*)^{V-1}] + 2\log[\frac{1}{V}\left(1-\frac{1}{V}\right)^{V-1}] \tag{20}$$

which is $\chi^2(1)$ under the null. Given the appropriate critical value, it is possible to derive a 95% confidence interval for TUFF of (6,439) for the 1% VaR and (-,87) for the 5% VaR[10]. The confidence intervals can be interpreted as follows. If VaR determined using a 1% significance level fails before the 6th observation, we can reject at the 5% level the null hypothesis that the model is adequate to cover losses on 99% of occasions. On the other hand, if the actual TUFF is greater than 439, then we would conclude that the model was leading to too high a value at risk, and therefore that the model was not failing as quickly as would be expected given the nominal 1% probability of failure.

It is perhaps worth noting that it is desirable from the point of view of the bank or securities firm concerned, for the calculated value at risk to be neither too large nor too small. A value at risk set too low could imply that the bank does not have sufficient capital to cover future losses, leading at best to regulatory scrutiny, and an increase in the scaling factor (resulting in a substantial increase in the capital requirement), and at worst to financial distress and possible company failure. Conversely, a VaR set too high, so that it covers more than the nominal percentage of horizons (e.g. an estimated 5% daily VaR which is actually sufficient to cover

99.9% of the out of sample periods), probably implies that the firm is tying up too much of its capital unnecessarily in an unprofitable fashion[11].

Whilst intuitive and simple to calculate, TUFF has obvious flaws as an evaluation metric. First, it is clearly not using much information from the sample, since all observations after the first failure are ignored, resulting in the test being over-sized. Thus, if the start of the out of sample period occurs at a time of exceptional market turbulence, a model which may have been perfectly adequate for the rest of the sample and incurring no further failures, would be rejected. Second, the TUFF statistic consequently has low power to reject models which are not adequate - this is clearly evidenced by the wide confidence intervals for TUFF presented above. For example, a 99% nominal coverage rate is expected to result in first failure at observation 100, but even if an exceedence of the VaR is recorded as early as observation 7, we cannot reject the underlying model at the 1% level; thus TUFF will have low ability to discriminate between volatility forecasts from different models.

Another simple method for determining model adequacy within the risk management framework is simply to calculate the percentage of times that the calculated value at risk is insufficient to cover the actual losses, during the rolling out of sample period. A good model would be one whose proportion of out of sample exceedences is close to the nominal value of (one minus coverage probability)% assumed (5% or 1%). We can also formulate a likelihood ratio test for the proportion of failures, in similar vein to (19) above. The probability of observing x failures in an actual sample of independent observations of size K will be distributed binomially, leading to the following test statistic distributed $\chi^2(1)$ under the null

$$UCF(K,x,p^*) = -2\log[(1-p^*)^{K-x}(p^*)^x] + 2\log[(1-\left(\frac{x}{K}\right))^{K-x}\left(\frac{x}{K}\right)^x] \qquad (21)$$

with notation as above. For ease of interpretation of the results, models are also ranked in the following way. We assume that any model which has a percentage of exceedences in the rolling hold-out sample which is greater than the nominal threshold, should be rejected as inadequate. Therefore, the lowest ranking models (classified as worst) are those which have the highest percentage of failures greater than the nominal value.

[10] It is not possible to establish a lower limit for the 5% VaR interval.
[11] Particularly in view of the regulatory scaling factor, which multiplies the firm's own value at risk estimate by at least 3.

When these models have been exhausted, we assume further that any model which generates far fewer exceedencess than the expected number is less desirable than a model which generates closer to the nominal number. Thus the best models under this loss function are those which generate less than, but closest to, the assumed coverage rate[12].

5. Results

5.1 Statistical Evaluation Criteria

The results for the volatility forecasts under standard statistical evaluation methods (percentage of over-predictions, mean squared error, and mean absolute error) are presented in Tables 3 to 6 for the government bond, FTA All-Share, commodities and portfolio series respectively.

Considering first the 1-step ahead (1 day) forecast horizon, a number of important features emerge. As one might anticipate, the random walk in volatility model produces roughly equal numbers of over-and under-predictions of realised volatility measured by the squared daily returns. On the other hand, all models over-predict volatility on average 70% of the time, except for the two EWMAs which over-predict more frequently than they under-predict. In all other respects, the random walk in volatility produces uniformly poor forecasts.

No clear "winners" emerge at the 1-day horizon, with different models being preferred for each series. MSE is clearly not a good discriminator at the top end, with many models ranked equally as the best. MAE, on the other hand, selects EWMA models for the bond and portfolio series, while for the share and commodity series, the GJR and autoregressive volatility models are preferred. In terms of the least accurate next-day forecasting models, the random walk in volatility and EGARCH models emerge as the worst performers, followed by the EWMAs for commodities and shares, although the latter proved the most accurate for the other two series.

An extension of the forecast and investment horizon to the one (trading) week, two week, and one month range does not markedly alter the relative model rankings, although the broad disagreement between criteria

[12] Of course, this could be replaced by a simpler symmetric or any other loss function if the user desired.

for a given series and model, is still apparent. For example, the autoregressive model, which ranks only seventh by MSE for the equities series at the one day horizon, ranks first when the investment horizon is extended to one month.

However, as Andersen and Bollerslev (1998) have shown, the use of low frequency squared returns is often not a useful way to evaluate volatility forecasts, and it is quite possible that when sums of higher-frequency squared returns are used instead as the *ex post* volatility measure, not only the values of the error measures, but also the model rankings, could change substantially. Thus for the 5, 10, and 20 day periods, we also evaluate the forecast accuracies using the sum of squared daily returns. Results are presented for the bond, share, commodities, and portfolio series in tables 7 to 10 respectively[13]. Comparing the results for the low frequency squared returns versus the high frequency sums of squared returns, we note firstly that the values of the error measures are as expected reduced considerably[14].

The GARCH model with *t*-distributed errors now emerges as the clear winner, producing the most accurate forecasts according to MAE, for three of the four series (bonds, stocks, and the portfolio). Only for the commodities return series does GARCH-*t* perform poorly. For the latter series, the long term mean and autoregressive volatility models prove to be the best under both squared and absolute error measures. Interestingly, the worst models seem invariant to both the use of a same-frequency or higher-frequency *ex post* measure, and to whether the errors are squared or the absolute values taken; a bad model appears to be a bad model whatever. Models which fit into this category are the random walk in volatility, the exponential GARCH, and the exponentially weighted moving average model.

5.2 Risk Management Evaluation Criteria

The corresponding evaluations for the forecasts when used in a risk management context are given in Tables 11 to 18. Volatility forecasts can be employed for the production of 99% and 95% nominal coverage rates for the value at risk estimates. In other words, forecasts are generated in respect of the amount of capital required to cover expected losses on 99% and 95% of days respectively. The results for these two sets of

[13] Of course, the results for the 1-step ahead evaluations will be identical to those of tables 3 to 6.
[14] Mean squared errors are reduced by roughly an order equivalent to the forecasting horizon, while absolute errors are reduced by a factor of around two for all horizons.

nominal coverage rates are provided in Tables 11 to 14 and 15 to 18 respectively for the 1 day, 1 week, 2 week and 1 month horizons. Three statistics are presented in each table - the time until first failure (TUFF), the proportion of failures (FT), and the test statistic associated with whether this proportion of failures is significantly higher than the nominal rate (UCF). Also given are the model rankings according to FT and UCF[15] as described in Section 4.3 above.

The first point to note is that if the objective is to cover 99% of future losses, then almost none of the models are adequate. The proportion of exceedences for the bond, share and commodity assets is always considerably in excess of 1% - typically 1.4% - 2%. Thus for example, even the best model at the 1-day horizon for the commodities data, which is the long term mean, has nearly 70% more violations of value at risk in the hold out sample than would be expected under the null. Also for this series, the majority of models have a TUFF statistic that takes on a value of one - that is, they fail at the first observation! Almost none of the models for any of the four asset classes makes it to the hundredth observation, the expected time until first failure. Consequently, the UCF statistic rejects all models for all individual asset series at all horizons.

Matters are improved somewhat for the portfolio of assets, presumably as a result of the benefits of diversification in reducing the number of extreme observations that lead to an exceedence of the VaR. The typical proportion of exceedences is reduced to around 1.2%, and although only the multivariate GARCH model has fewer than 1% exceedences, several models are acceptable according to the UCF test statistic. Similar patterns are revealed at the 1-day and longer horizons. The models fare much better when only 95% coverage is desired; more than half of the models achieve their nominal rate. In terms of model rankings, the long term mean and the linear regression in volatility models seem preferable, although again, there is no uniformly most accurate model. The GARCH model seems to provide reasonably accurate VaR estimates, evidenced by its actual coverage rate being close to the nominal rate, although there is a tendency to over-estimate the VaR, a result also observed by Brooks, Clare and Persand (2000).

6. Conclusions

[15] The rankings according to FT and UCF will of course by definition be identical.

This paper has sought to re-examine the volatility forecasting literature in the context of a relatively new use of volatility forecasts - for financial (market) risk assessment. A number of our results are worthy of further note. First, the gain from using a multivariate GARCH model for forecasting volatility, which has not been previously investigated, is minimal. This result is true both under standard statistical and risk management evaluation measures. Given the complexity, estimation difficulties, and computer-intensive nature of MGARCH modelling, we conjecture that unless the conditional covariances are required, the estimation of mutlivariate GARCH models is not worthwhile. In the context of portfolio volatility, more accurate results can be obtained by aggregating the portfolio constituents into a single series, and forecasting that, than modelling the individual component volatilities and the correlations between the returns.

Second, it appears that some models are poor performers irrespective both of the series on which they are estimated, and the loss function used to evaluate their forecasts. The random walk in volatility, the EGARCH and to a lesser extent the EWMA models, fall into this category.

When it comes to selecting the "best" model for forecasting, however, the particular evaluation measure employed plays a predominant role. Whilst there seems to be little difference in the model rankings when the *ex post* measure is changed from low-frequency to high-frequency squared returns, the differences between rankings under statistical and risk management procedures are substantial. Although generalising across data series (asset classes) and investment horizons is difficult, overall the statistical measures preferred the GARCH(1,1) model over simpler techniques and over its extensions and variants. On the other hand, when evaluated in the context of VaR estimates which achieve an appropriate out of sample coverage rate, the simplest models, such as the long term mean (historical average) or the autoregressive volatility model, are preferred. We thus concur with Dacco and Satchell (1999) in arguing that the choice of loss function can have an over-riding effect upon volatility forecasting accuracies; thus the debate on superior volatility forecasting models should be considered far from resolved.

References

Akgiray, V. Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts Journal of Business **62** (1989), 55-80.

Alexander, C.O. and Leigh, C.T. On the Covariance Models used in Value at Risk Models, Journal of Derivatives, **4** (1997), 50-62.

Andersen, T. and Bollerslev, T. Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts International Economic Review **39** (1998), 885-905

Basle Committee on Banking Supervision July, International Convergence of Capital Measurement and Capital Standards (1988)

Brailsford, T.J. and Faff, R.W. An Evaluation of Volatility Forecasting Techniques Journal of Banking and Finance **20** (1996), 419-438

Bollerslev, T., Chou, R.Y. and Kroner, K.F. ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence Journal of Econometrics **52** (1992), 5-59

Bollerslev, T., Engle, R.F. and Wooldridge, J.M. A Capital Asset Pricing Model with Time-Varying Covariances Journal of Political Economy **96** (1988), 116-131

Brooks, C. Forecasting Stock Return Volatility: Does Volume Help? Journal of Forecasting **17** (1998), 59-80

Brooks, C., Clare, A.D., and Persand, G. A Word of Caution on Calculating Market-Based Minimum Capital Risk Requirements Journal of Banking and Finance **14(10)**, (2000), 1557-1574

Brooks, C., Henry, O.T., and Persand, G. Optimal Hedging and the Value of News forthcoming Journal of Business

Brooks, C. and Persand, G. Value at Risk and Market Crashes Journal of Risk **2(4)**, (2000), 5-26

Brooks, C., and Persand, G. Lies, Damned Lies and Value at Risk Estimates Risk **13(5) May** (2000), 63-66

Dacco, R. and Satchell, S. Why do Regime-Switching Models Forecast So Badly? Journal of Forecasting **18** (1999), 1-16

Day, T.E. and Lewis, C.M. Stock Market Volatility and the Information Content of Stock Index Options Journal of Econometrics **52** (1992), 267-287

Dowd, K. Beyond Value at Risk: The New Science of Risk Management (1998) Wiley, Chichester, UK

Embrechts, P., Resnick, S.I. and Samorodnitsky, G. Extreme Value Theory as a Risk Management Tool North American Actuarial Journal **3** (1999), 30-41

Fair, R.C. and Shiller, R.J. 'Comparing Information in Forecasts from Econometric Models', American Economic Review **80** (1990), 375-389

Franses, P.H. and van Dijk, D. Forecasting Stock Market Volatility Using Non-Linear Garch Models Journal of Forecasting **15** (1996), 229-235

Heynen, R.C. and Kat, H.M. Volatility Prediction: A Comparison of the Stochastic Volatility, GARCH(1,1) and EGARCH(1,1) Models Journal of Derivatives **2** (1994), 50-65

Huisman, R., Koedijk, K.G., and Poqwnall, R.A.J. VaR-x: Fat Tails in Financial Risk Management Journal of Risk **1** (1998), 47-61

Jackson, P., Maude, D.J., and Perraudin, W. Testing Value at Risk Approaches to Capital Adequacy Bank of England Quarterly Bulletin, **38** (1998), 256-266

Johansen, A., and Sornette, D. Critical Crashes Risk **12** (1999), 91-95

Jorion, P. Value at Risk: The New Benchmark for Controlling Market Risk, (1996) Chicago: Irwin.

J.P. Morgan Riskmetrics Technical Document (1996), 4th Edition.

Kupiec, P. Techniques for Verifying the Accuracy of Risk Measurement Models Journal of Derivatives **2** (1995), 73-84

Pagan, A.R. and Schwert, G.W. Alternative Models for Conditional Stock Volatilities Journal of Econometrics **45** (1990), 267-290

West, K.D. and Cho, D. The Predictive Ability of Several Models of Exchange Rate Volatility Journal of Econometrics **69** (1995), 367-391

West, K.D., Edison, H.J. and Cho, D. A Utility-Based Comparison of some Models of Exchange Rate Volatility Journal of International Economics **35** (1993), 23-45

Table 1
Summary Statistics

	Long Govt Bond	FTSE All Share	Reuters Commodities	Portfolio
Mean	0.000233	0.000301	-0.000219	0.000171
Variance	6.50×10^{-6}	1.410×10^{-5}	6.210×10^{-6}	3.691×10^{-6}
Skewness	0.0132	-1.063**	-0.5663**	-0.291**
Kurtosis	3.37**	14.654**	18.369**	4.446**
Bera-Jarque Statistic	2300**	44400**	68700**	4080**

Notes: The Bera Jarque statistic is distributed asymptotically as a $\chi^2(2)$ under the null of normality. * and ** indicate significance at the 5% and 1% levels respectively.

Table 2: Description of Models Used for Forecasting

Model	Arcronym	Equations for Model	Equation #
1. Random walk in volatility	RW	$\sigma^2_{f,t+n} = \sigma^2_t$	(2)
2. Long term mean	LTM	$\sigma^2_{f,t+n} = \frac{1}{1250}\sum_{j=t-1249}^{t}\sigma^2_{t-j}$	(3)
3. Short term moving average	MA5	$\sigma^2_{f,t+n} = \frac{1}{5}\sum_{j=0}^{4}\sigma^2_{t-j}$	(4)
4. Long term moving average	MA100	$\sigma^2_{f,t+n} = \frac{1}{100}\sum_{j=0}^{99}\sigma^2_{t-j}$	(5)
5. Linear regression with 1 lag	AR1	$\sigma^2_{f,t+n} = \alpha_0 + \alpha_1\sigma^2_t + \varepsilon_t$	(6)
6. Linear regression with AIC lags	ARAIC	$\sigma^2_{f,t+n} = \beta_0 + \sum_{j=0}^{p-1}\beta_j\sigma^2_{t-j} + \varepsilon_t$	(7)
7. GARCH(1,1)	GAR	$r_{t+1} = \mu + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma^2_{t+1}),$ $\sigma^2_{f,t+n} = \gamma_0 + \varphi_1\varepsilon^2_t + \gamma_2\sigma^2_t$	(8)
8. GJR(1,1)	GJR	$\sigma^2_{f,t+n} = \delta_0 + \delta_1\varepsilon^2_t + \delta_2\sigma^2_t + \delta_3 S^-_t\varepsilon^2_t$ S^-_t =1for $\varepsilon_t \leq 0$ and 0 otherwise	(9)
9. EGARCH(1,1)	EGAR	$\log(\sigma^2_{f,t+n}) = \omega_1 + \omega_2\log(\sigma^2_t) + \omega_3\frac{\varepsilon_t}{\sqrt{\sigma^2_t}}$ $+\omega_4\left[\frac{\lvert\varepsilon_t\rvert}{\sqrt{\sigma^2_t}} - \sqrt{\frac{2}{\pi}}\right]$	(10)
10. Long exponentially weighted moving average	EMA5	$\sigma^2_{f,t+n} = (1-\lambda_1)\sum_{t=1}^{5}\lambda_1^{t-1}(r_t - \bar{r})$	(11)
11. Short exponentially weighted moving average	EMA100	$\sigma^2_{f,t+n} = (1-\lambda_1)\sum_{t=1}^{100}\lambda_1^{t-1}(r_t - \bar{r})$	(12)
12. GARCH with *t*-distributed errors	GART	$r_{t+1} = \mu + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim t_k(0, \sigma^2_{t+1}),$ $\sigma^2_{f,t+n} = \gamma_0 + \varphi_1\varepsilon^2_t + \gamma_2\sigma^2_t$	(13)
13. Multivariate GARCH	MGAR	See text for model description.	-

Notes: Forecast equations are given for $n = 1$ step ahead, and recursions can easily be computed from these for the 2,3,…,20 step ahead forecasts. The model order p for ARAIC is determined individually for each forecast iteration by the minimisation of Akaike's information criterion, with maximal lag 5. All model parameters are estimated using quasi-maximum likelihood. The exponentially weighted moving average coefficients (λ_i) are chosen to produce the best fit by minimising the sum of the squared in-sample forecast errors.

Table 3: Statistical Loss Functions for *Government Bond* (Same frequency squared returns as *ex post* measure)

Steps	***1***						***5***						***10***						***20***					
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	49.88	1	0.036	12	0.082	12	45.31	2	0.927	13	0.425	13	43.09	3	3.146	13	0.854	13	41.59	5	14.25	13	1.903	13
LTM	76.97	13	0.020	1=	0.069	9=	70.82	11	0.485	3=	0.337	10	69.35	11	1.357	3	0.673	6=	67.12	10	6.362	1=	1.494	6
MA5	67.01	3	0.021	8	0.068	7=	59.36	3	0.545	9	0.352	11	57.02	4	1.693	10	0.717	11	55.74	3	7.923	10	1.617	12
MA100	73.63	6	0.020	1=	0.066	3=	67.23	9	0.485	3=	0.329	5=	64.34	9	1.406	5	0.673	6=	62.42	7	6.777	4	1.515	9
AR1	76.44	11=	0.020	1=	0.069	9=	70.90	12	0.484	2	0.336	9	69.37	12	1.354	2	0.672	5	67.15	11	6.362	1=	1.493	5
ARAIC	75.22	10	0.020	1=	0.068	7=	69.76	10	0.481	1	0.334	7=	68.73	10	1.349	1	0.668	4	66.98	9	6.393	3	1.495	7=
GAR	73.85	7	0.020	1=	0.066	3=	65.95	7	0.489	6	0.327	4	60.89	5	1.440	4	0.661	3	57.33	4	7.071	6	1.482	4
GJR	76.44	11=	0.024	10=	0.076	11	62.78	5	0.547	10	0.343	2	54.35	1	1.565	8	0.674	8=	46.57	2	7.618	8	1.480	2
EGAR	69.96	5	0.170	13	0.135	13	48.01	1	0.796	12	0.412	12	37.83	6	2.000	12	0.772	12	28.76	12	8.324	11	1.640	10
EMA5	43.48	2	0.022	9	0.056	1	37.89	4	0.542	8	0.302	1	35.86	8	1.628	9	0.627	1	34.38	8	7.870	9	1.440	1
EMA100	31.77	4	0.024	10=	0.061	2	26.87	13	0.597	11	0.334	7=	25.76	13	1.860	11	0.702	10	24.53	13	8.765	12	1.576	11
GART	74.16	8	0.020	1=	0.066	3=	67.20	8	0.485	3=	0.329	5=	63.42	7	1.430	6	0.674	8=	60.33	6	6.920	5	1.495	7=
MGAR	74.83	9	0.020	1=	0.067	6	63.67	6	0.496	7	0.321	3	56.58	2	1.486	7	0.654	2	51.02	1	7.418	7	1.481	3

Table 4: Statistical Loss Functions for *Equities* (Same frequency squared returns as *ex post* measure)

Steps	***1***						***5***						***10***						***20***					
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	50.21	1	0.420	9=	0.175	11	45.54	1	18.26	13	1.032	12	43.14	6	103.2	13	2.308	13	41.03	6	440.2	13	5.344	13
LTM	75.19	13	0.415	7=	0.156	9=	69.01	12	15.02	6	0.906	8=	65.90	12	78.87	3	1.925	9	62.39	9	313.3	4	4.442	9
MA5	63.95	3	0.389	5	0.148	3	57.89	4	15.26	9	0.904	7	54.88	4	79.81	8	1.913	8	50.38	1	315.9	5	4.333	4
MA100	69.37	6	0.412	6	0.156	9=	63.03	9	15.07	7	0.906	8=	60.25	7	79.38	4	1.977	11	56.44	5	313.0	2=	4.352	5
AR1	71.85	11	0.387	4	0.151	5	67.87	11	14.57	1	0.886	5	65.81	11	78.21	1	1.908	7	62.20	8	313.0	2=	4.432	8
ARAIC	70.32	8	0.415	7=	0.153	7	67.01	10	14.97	4	0.890	6	65.48	10	78.77	2	1.907	6	62.11	7	312.7	1	4.427	7
GAR	70.68	9	0.342	1	0.142	2	61.25	8	14.93	3	0.842	3	55.36	5	79.47	6	1.808	3	44.70	4	318.9	7	4.275	1
GJR	72.05	12	0.420	9=	0.149	4	57.52	3	15.13	8	0.830	1	46.98	1	80.05	9	1.796	2	34.94	10	323.8	10	4.363	6
EGAR	66.15	4	0.796	12	0.274	12	45.40	2	15.54	10	0.929	11	36.97	8	80.81	10	1.984	12	30.63	12	323.5	9	4.610	12
EMA5	43.98	2	2.859	13	0.443	13	39.50	6	17.38	12	1.147	13	35.10	9	80.89	11	1.897	5	32.63	11	324.1	11	4.517	10
EMA100	32.18	5	0.475	11	0.155	8	29.35	13	15.91	11	0.922	10	26.98	13	81.37	12	1.943	10	24.56	13	325.7	12	4.533	11
GART	70.18	7	0.343	2	0.139	1	60.83	7	14.90	2	0.834	2	54.74	3	79.52	7	1.793	1	44.98	3	319.8	8	4.286	2
MGAR	71.74	10	0.370	3	0.152	6	60.20	5	15.01	5	0.857	4	54.24	2	79.42	5	1.850	4	45.70	2	317.2	6	4.321	3

Table 5: Statistical Loss Functions for *Commodities* (Same frequency squared returns as *ex post* measure)

Steps	***1***						***5***						***10***						***20***					
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	50.07	1	0.146	11	0.091	10	45.40	5	2.683	13	0.446	12	43.59	2	10.78	13	0.956	12	40.95	4	45.63	13	2.124	13
LTM	78.25	13	0.082	1=	0.076	2=	73.63	13	0.831	3	0.334	7	69.85	10	3.086	4	0.705	6=	66.31	8	16.17	4	1.575	5=
MA5	67.23	4	0.095	7=	0.084	8=	61.53	7	1.049	9	0.369	9	59.11	5	3.736	9	0.781	11	55.58	2	17.51	7	1.712	10
MA100	76.97	8	0.083	5	0.079	5=	72.13	10	0.835	4	0.345	8	68.71	9	3.076	1	0.721	8	64.23	5	15.89	1	1.616	8
AR1	77.41	11	0.082	1=	0.075	1	73.35	11	0.830	2	0.333	5=	69.96	13	3.085	3	0.705	6=	66.20	6	16.16	3	1.576	7
ARAIC	77.25	9	0.082	1=	0.076	2=	73.46	12	0.829	1	0.333	5=	69.90	11=	3.084	2	0.704	5	66.23	7	16.14	2	1.575	5=
GAR	76.55	6	0.095	7=	0.081	7	54.94	6	0.894	8	0.296	3	41.98	3	3.349	7	0.641	2	29.07	10	17.85	8	1.534	1
GJR	77.75	12	0.108	9	0.084	8=	46.20	4	0.887	7	0.290	2	34.44	8	3.341	6	0.643	3	23.84	13	17.86	9	1.553	3
EGAR	72.63	5	0.344	12	0.156	13	53.10	3	1.472	10	0.430	10	41.64	4	3.783	10	0.751	9	32.85	9	18.28	11	1.635	9
EMA5	55.97	2	0.124	10	0.115	11	50.54	1	1.796	11	0.530	13	47.84	1	5.455	12	0.993	13	44.40	3	16.68	5	1.785	12
EMA100	37.36	3	0.353	13	0.125	12	31.15	9	1.838	12	0.434	11	30.10	11=	3.786	11	0.758	10	27.65	11	18.87	12	1.713	11
GART	77.27	10	0.082	1=	0.077	4	51.41	2	0.865	5	0.284	1	39.03	7	3.374	8	0.639	1	26.29	12	18.00	10	1.546	2
MGAR	76.80	7	0.093	6	0.079	5=	66.68	8	0.881	6	0.330	4	60.42	6	3.289	5	0.697	4	51.43	1	17.07	6	1.565	4

Table 6: Statistical Loss Functions for *Portfolio* (Same frequency squared returns as *ex post* measure)

Steps	***1***						***5***						***10***						***20***					
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	49.88	1	0.036	12	0.082	12	47.57	1	0.399	13	0.245	12	44.84	3	1.637	13	0.520	12	42.17	3	6.637	13	1.144	12
LTM	76.97	13	0.020	1=	0.069	9=	70.40	10	0.200	1	0.196	7	68.46	10	0.756	1	0.408	6	62.87	7	2.927	1	0.900	4=
MA5	67.01	3	0.021	8	0.068	7=	60.81	4	0.259	10	0.209	11	58.03	4	0.955	10	0.441	10	53.21	1	3.420	8	0.969	9
MA100	73.63	6	0.020	1=	0.066	3=	66.98	7	0.201	2	0.193	6	63.89	7	0.764	2	0.405	5	58.72	4	3.035	4	0.914	8
AR1	76.44	11=	0.020	1=	0.069	9=	69.65	9	0.212	5	0.197	8=	68.26	9	0.772	3	0.409	7	62.92	8	2.950	3	0.902	6
ARAIC	75.22	10	0.020	1=	0.068	7=	69.40	8	0.218	6=	0.197	8=	68.01	8	0.777	4	0.410	8	62.70	6	2.930	2	0.900	4=
GAR	73.85	7	0.020	1=	0.066	3=	60.89	5	0.210	4	0.182	3=	51.63	1	0.823	7	0.387	4	40.39	5	3.399	7	0.894	3
GJR	76.44	11=	0.024	10=	0.076	11	54.08	2	0.224	8	0.161	1	39.94	5	0.784	5	0.327	1	28.10	10	3.064	5	0.795	1
EGAR	69.96	5	0.170	13	0.135	13	44.70	3	0.218	6=	0.171	2	36.16	6	0.864	8	0.380	2	29.79	9	3.581	10	0.907	7
EMA5	43.48	2	0.022	9	0.056	1	28.43	11	0.231	9	0.191	5	25.40	11	0.900	9	0.415	9	22.03	12	3.700	11	0.971	10
EMA100	31.77	4	0.024	10=	0.061	2	20.22	12	0.289	12	0.203	10	18.47	12	1.164	12	0.445	11	15.94	13	3.888	12	0.994	11
GART	74.16	8	0.020	1=	0.066	3=	62.87	6	0.205	3	0.182	3=	54.27	2	0.802	6	0.383	3	43.39	2	3.334	6	0.885	2
MGAR	74.83	9	0.020	1=	0.067	6	87.82	13	0.282	11	0.346	13	83.20	13	0.997	11	0.659	13	75.74	11	3.489	9	1.261	13

Table 7: Statistical Loss Functions for *Government Bond* (Sum of Daily Squared Returns as *ex post* measure)

Steps			**1**						**5**						**10**						**20**			
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	49.88	1	0.036	12	0.082	12	35.27	6	0.553	13	0.344	13	32.18	7	2.052	13	0.660	13	30.40	9	8.042	13	1.308	13
LTM	76.97	13	0.020	1=	0.069	9=	71.07	10	0.143	7	0.232	8	68.65	9	0.376	6	0.406	8	68.29	6	1.042	5	0.725	7
MA5	67.01	3	0.021	8	0.068	7=	51.18	1	0.181	8	0.231	7	46.62	2	0.561	10	0.407	9	43.26	3	1.716	9	0.788	9
MA100	73.63	6	0.020	1=	0.066	3=	64.17	5	0.137	5	0.214	4	61.14	5	0.359	4	0.369	4	60.67	4	1.016	3	0.667	3
AR1	76.44	11=	0.020	1=	0.069	9=	71.10	11	0.141	6	0.230	6	68.51	8	0.372	5	0.403	7	68.32	7	1.036	4	0.722	6
ARAIC	75.22	10	0.020	1=	0.068	7=	70.38	9	0.136	4	0.222	5	68.82	10	0.354	3	0.387	5	68.35	8	1.001	1	0.705	5
GAR	73.85	7	0.020	1=	0.066	3=	62.45	4	0.130	2	0.201	2	56.05	3	0.347	2	0.341	1	47.43	1	1.077	6	0.633	2
GJR	76.44	11=	0.024	10=	0.076	11	57.41	2	0.186	10	0.233	9	43.87	4	0.472	8	0.401	6	29.74	10	1.495	8	0.761	8
EGAR	69.96	5	0.170	13	0.135	13	32.10	8	0.421	12	0.333	12	20.47	11	0.908	12	0.603	12	15.08	11	2.690	12	1.180	12
EMA5	43.48	2	0.022	9	0.056	1	18.89	12	0.184	9	0.237	10	14.63	12	0.544	9	0.458	10	12.74	12	1.737	10	0.896	10
EMA100	31.77	4	0.024	10=	0.061	2	9.040	13	0.228	11	0.277	11	6.898	13	0.731	11	0.549	11	6.370	13	2.535	11	1.093	11
GART	74.16	8	0.020	1=	0.066	3=	65.31	7	0.128	1	0.204	3	61.25	6	0.336	1	0.345	2	55.16	2	1.008	2	0.628	1
MGAR	74.83	9	0.020	1=	0.067	6	58.80	3	0.135	3	0.200	1	47.90	1	0.377	7	0.347	3	35.74	5	1.204	7	0.671	4

Table 8: Statistical Loss Functions for *Equities*(Sum of Daily Squared Returns as *ex post* measure)

Steps			**1**						**5**						**10**						**20**			
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	50.21	1	0.420	9=	0.175	11	36.47	6	9.775	13	0.774	12	34.72	6	40.40	13	1.550	13	33.02	4	161.3	13	3.147	13
LTM	75.19	13	0.415	7=	0.156	9=	72.16	11	4.833	7	0.582	9	70.77	7	13.00	6	1.065	9	71.99	8	34.24	7	2.001	8
MA5	63.95	3	0.389	5	0.148	3	51.18	1	4.898	9	0.522	5	46.43	4	13.17	8	0.937	5	41.47	1	33.72	3	1.734	4
MA100	69.37	6	0.412	6	0.156	9=	64.03	7	4.891	8	0.541	7	62.75	5	13.28	9	0.991	7	58.67	2	32.30	1	1.657	3
AR1	71.85	11	0.387	4	0.151	5	71.10	8	4.614	1	0.552	8	70.63	8	12.62	1	1.029	8	71.57	6	33.84	4	1.961	7
ARAIC	70.32	8	0.415	7=	0.153	7	71.29	9	4.729	4	0.531	6	71.82	10	12.69	3	0.985	6	71.79	7	33.67	2	1.899	6
GAR	70.68	9	0.342	1	0.142	2	59.39	5	4.677	3	0.450	2	47.96	1	12.67	2	0.789	1	26.90	9	34.12	5	1.587	1
GJR	72.05	12	0.420	9=	0.149	4	51.41	2	4.773	6	0.464	3	29.93	9	13.10	7	0.888	4	6.65	12	36.79	9	2.031	9
EGAR	66.15	4	0.796	12	0.274	12	27.96	10	5.257	10	0.678	11	20.75	11	14.02	11	1.275	12	17.58	10	37.93	11	2.495	12
EMA5	43.98	2	2.859	13	0.443	13	20.83	12	7.328	12	0.867	13	14.08	12	13.88	10	1.145	10	12.02	11	37.82	10	2.270	10
EMA100	32.18	5	0.475	11	0.155	8	10.35	13	5.540	11	0.663	10	7.455	13	14.06	12	1.204	11	6.175	13	38.01	12	2.369	11
GART	70.18	7	0.343	2	0.139	1	59.05	4	4.674	2	0.443	1	47.34	3	12.79	5	0.792	2	28.93	5	34.35	8	1.603	2
MGAR	71.74	10	0.370	3	0.152	6	58.47	3	4.743	5	0.475	4	47.84	2	12.74	4	0.867	3	36.89	3	34.16	6	1.741	5

Table 9: Statistical Loss Functions for *Commodities* (Sum of Daily Squared Returns as *ex post* measure)

Steps	***1***						***5***						***10***						***20***					
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	50.07	1	0.146	11	0.091	10	34.55	7	2.425	13	0.423	12	30.52	4	8.999	13	0.828	12	27.40	7	34.21	13	1.630	13
LTM	78.25	13	0.082	1=	0.076	2=	73.83	12	0.508	1=	0.268	5=	71.13	7	1.059	1	0.466	3	68.57	4	2.293	3	0.802	3
MA5	67.23	4	0.095	7=	0.084	8=	51.13	1	0.735	9	0.320	9	45.31	2	1.739	10	0.587	9	40.72	1	3.746	9	1.044	6
MA100	76.97	8	0.083	5	0.079	5=	68.23	8	0.523	4	0.289	8	64.70	3	1.120	4	0.508	6	60.36	2	2.557	4	0.890	5
AR1	77.41	11	0.082	1=	0.075	1	73.27	9=	0.508	1=	0.267	4	71.24	8	1.060	2=	0.465	1=	68.65	5	2.292	2	0.801	2
ARAIC	77.25	9	0.082	1=	0.076	2=	73.27	9=	0.508	1=	0.268	5=	71.10	6	1.060	2=	0.465	1=	68.76	6	2.291	1	0.800	1
GAR	76.55	6	0.095	7=	0.081	7	41.25	2	0.586	8	0.255	2	16.63	10	1.306	6	0.501	5	3.143	12	3.378	6	1.086	7
GJR	77.75	12	0.108	9	0.084	8=	26.70	11	0.575	6=	0.260	3	7.677	13	1.312	7	0.542	8	2.337	13	3.472	8	1.152	9
EGAR	72.63	5	0.344	12	0.156	13	36.33	4	1.156	10	0.396	10	19.94	9	1.758	11	0.634	10	11.60	9	3.902	10	1.195	10
EMA5	55.97	2	0.124	10	0.115	11	35.13	5	1.556	12	0.519	13	29.46	5	3.817	12	0.891	13	24.42	8	4.358	12	1.329	12
EMA100	37.36	3	0.353	13	0.125	12	14.47	13	1.510	11	0.421	11	11.27	12	1.688	9	0.655	11	10.13	10	4.208	11	1.250	11
GART	77.27	10	0.082	1=	0.077	4	34.99	6	0.550	5	0.249	1	13.07	11	1.313	8	0.520	7	4.534	11	3.456	7	1.113	8
MGAR	76.80	7	0.093	6	0.079	5=	61.31	3	0.575	6=	0.271	7	50.04	1	1.240	5	0.477	4	38.61	3	2.927	5	0.881	4

Table 10: Statistical Loss Functions for *Portfolio* (Sum of Daily Squared Returns as *ex post* measure)

Steps	***1***						***5***						***10***						***20***					
Models	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*	*%OP*	*Rank*	*MSE*	*Rank*	*MAE*	*Rank*
RW	49.88	1	0.012	9=	0.045	11	35.86	5	0.257	13	0.199	12	33.10	4	0.997	13	0.388	12	31.43	6	3.850	13	0.765	12
LTM	74.91	10	0.009	1=	0.038	7=	71.10	9	0.088	2	0.123	6	69.04	7=	0.208	1	0.212	6	68.10	5	0.494	2	0.368	3
MA5	65.09	3	0.010	4=	0.039	9	51.07	2	0.136	10	0.140	9	46.62	1	0.379	10	0.250	7	43.53	1	0.837	9	0.462	7
MA100	71.54	5	0.009	1=	0.037	4=	64.62	6	0.090	3	0.118	3	63.17	3	0.217	4	0.206	3	60.06	2	0.543	4	0.371	4
AR1	72.35	9	0.010	4=	0.038	7=	69.68	8	0.093	4	0.122	5	68.99	6	0.214	2	0.210	5	67.73	4	0.495	3	0.364	2
ARAIC	71.93	6=	0.018	11	0.040	10	69.21	7	0.097	6	0.121	4	69.04	7=	0.216	3	0.209	4	67.37	3	0.490	1	0.362	1
GAR	72.21	8	0.010	4=	0.037	4=	50.77	1	0.095	5	0.109	2	32.99	5	0.242	6	0.203	2	14.35	9	0.669	6	0.439	6
GJR	75.05	11	0.044	13	0.046	12	36.41	4	0.111	8	0.137	8	22.31	9	0.275	8	0.269	9	16.36	8	0.770	7	0.538	8
EGAR	60.33	2	0.010	4=	0.033	1	17.86	10	0.100	7	0.125	7	4.089	12	0.273	7	0.261	8	0.362	13	0.788	8	0.548	9
EMA5	30.71	4	0.010	4=	0.035	2	8.567	11	0.112	9	0.159	10	5.953	11	0.309	9	0.317	10	5.007	11	0.917	10	0.636	10
EMA100	22.09	12	0.012	9=	0.036	3	4.367	13	0.163	11	0.175	11	3.282	13	0.531	12	0.352	11	2.976	12	0.964	11	0.655	11
GART	71.93	6=	0.009	1=	0.037	4=	54.13	3	0.087	1	0.107	1	37.58	2	0.219	5	0.191	1	16.94	7	0.612	5	0.407	5
MGAR	91.32	13	0.032	12	0.081	13	94.55	12	0.173	12	0.299	13	91.29	10	0.484	11	0.525	13	85.79	10	1.276	12	0.910	13

Table 11: ***Government Bond (Risk Management Evaluation - 1% VaR)***

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	3	12.41	640.7	12	1	11.77	588.6	12	1	10.96	524.6	10	1	9.990	450.1	11
LTM	33	1.613	5.004	7=	396	0.967	0.011	1	437	0.974	0.011	1=	429	1.168	0.424	1=
MA5	23	3.255	50.33	9	98	3.227	49.29	9	94	2.726	31.94	8	3	3.004	41.25	7
MA100	23	1.530	3.811	4=	22	1.307	1.359	4	22	1.335	1.603	5	429	1.474	3.095	4
AR1	33	1.613	5.004	7=	397	0.946	0.047	2	437	0.974	0.011	1=	429	1.168	0.424	1=
ARAIC	33	1.530	3.811	4=	397	1.196	0.571	3	437	0.946	0.047	3	429	1.196	0.571	3
GAR	23	1.502	3.445	2=	22	1.586	4.590	6	267	1.808	8.309	6	428	2.281	18.99	6
GJR	23	1.446	2.762	1	22	2.782	33.73	8	397	3.922	77.49	9	14	5.814	172.9	9
EGAR	121	3.394	55.65	10	20	8.734	359.3	10	6	12.13	617.9	12	3	14.52	821.2	12
EMA5	15	11.21	544.2	11	20	11.66	579.6	11	21	11.49	566.3	11	4	9.847	439.7	10
EMA100	3	18.55	1195	13	1	18.05	1147	13	1	18.11	1152	13	1	16.16	969.0	13
GART	23	1.502	3.445	2=	22	1.335	1.603	5	434	1.224	0.738	4	14	1.558	4.193	5
MGAR	23	1.558	4.193	6	21	2.031	12.90	7	21	2.420	22.76	7	90	3.727	69.16	8

Table 12: ***Equities (Risk Management Evaluation 1% VaR)***

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	9	11.79	590.8	12	30	10.99	526.7	12	44	10.24	468.9	10	64	9.708	429.4	10
LTM	444	1.697	6.334	7	440	1.752	7.293	2	713	1.502	3.445	3	703	1.975	11.68	3
MA5	44	2.420	22.76	9	122	3.700	67.99	9	397	3.477	58.93	7	117	3.978	79.93	5
MA100	23	1.641	5.432	5=	30	2.197	16.86	4	713	1.725	6.806	4	703	2.197	16.86	4
AR1	23	1.641	5.432	5=	440	1.780	7.794	3	713	1.446	2.762	2	703	1.947	11.08	2
ARAIC	397	1.780	7.794	8	440	1.613	5.004	1	713	1.252	0.925	1	703	1.780	7.794	1
GAR	23	1.391	2.148	2	132	2.420	22.76	5	397	2.865	36.48	6	118	4.840	120.8	6
GJR	23	1.419	2.446	3	30	3.366	54.57	8	126	4.590	108.4	9	109	7.594	281.8	9
EGAR	16	3.115	45.20	10	30	8.651	353.4	10	44	13.05	694.0	12	65	15.83	938.5	12
EMA5	23	10.79	511.6	11	30	11.13	537.6	11	119	12.10	615.6	11	114	10.85	515.9	11
EMA100	16	17.16	1062	13	30	17.25	1070	13	68	17.83	1125	13	108	16.33	984.5	13
GART	23	1.474	3.095	4	132	2.531	25.96	6	397	2.698	31.06	5	118	5.007	129.3	7
MGAR	23	1.307	1.359	1	30	2.587	27.62	7	121	3.533	61.15	8	109	6.203	195.4	8

Table 13: *Commodities (Risk Management Evaluation 1% VaR)*

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	1	15.02	865.6	12	1	16.69	1018	12	55	17.39	1083	12	45	20.17	1355	9
LTM	1	1.697	6.334	1	36	1.280	1.132	1=	67	1.725	6.806	1	59	2.392	21.99	1
MA5	36	4.312	95.08	10	36	4.256	92.49	6	61	5.035	130.7	5	53	7.510	276.4	5
MA100	1	2.114	14.83	8	36	1.864	9.379	4	69	2.142	15.49	4	355	2.615	28.47	4
AR1	1	1.864	9.379	2=	36	1.280	1.132	1=	67	1.752	7.293	2=	59	2.420	22.76	2=
ARAIC	1	1.864	9.379	2=	36	1.280	1.132	1=	67	1.752	7.293	2=	59	2.420	22.76	2=
GAR	1	2.003	12.28	6	36	6.231	197.0	7	60	13.41	724.7	8	50	22.84	1630	10
GJR	36	1.947	11.08	5	36	8.679	355.4	9	36	16.86	1033	11	36	26.15	1988	12
EGAR	36	3.282	51.38	9	36	8.790	363.2	10	36	14.38	808.9	9	36	22.06	1548	8
EMA5	1	9.513	415.1	11	36	10.85	515.9	11	36	12.71	666.1	7	36	15.27	888.0	7
EMA100	1	19.69	1307	13	36	23.39	1689	13	36	24.45	1802	13	36	26.26	2001	13
GART	1	1.919	10.50	4	36	6.843	233.9	8	36	14.99	863.1	10	36	24.42	1799	11
MGAR	36	2.031	12.90	7	36	3.255	50.33	5	36	5.480	154.4	6	36	8.734	359.3	6

Table 14: *Portfolio (Risk Management Evaluation 1% VaR)*

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	12	12.24	626.9	11	12	11.63	577.4	11	36	10.74	507.3	10	36	10.24	468.9	9
LTM	374	1.140	0.298	2=	370	1.113	0.193	2	367	0.946	0.047	1=	359	1.140	0.298	1
MA5	16	3.032	42.22	9	75	3.171	47.23	8	132	2.893	37.42	6	117	3.227	49.29	6
MA100	23	1.502	3.445	8	370	1.168	0.424	5	368	1.252	0.925	5	359	1.391	2.148	5
AR1	23	1.335	1.603	6	370	1.140	0.298	3	367	0.946	0.047	1=	359	1.168	0.424	2=
ARAIC	36	1.474	3.095	7	370	1.224	0.738	4	367	0.918	0.109	3	359	1.168	0.424	2=
GAR	23	1.140	0.298	2=	370	2.448	23.55	7	365	3.588	63.41	8	122	6.704	225.4	8
GJR	374	1.140	0.298	2=	30	7.566	280.0	10	44	11.27	548.6	11	60	16.30	981.9	12
EGAR	23	3.672	66.84	10	30	7.510	276.4	9	68	8.039	311.4	9	68	10.40	481.6	10
EMA5	16	18.19	1160	12	29	16.83	1031	12	36	15.83	938.5	12	116	15.91	946.1	11
EMA100	3	25.06	1869	13	29	22.69	1615	13	36	22.73	1618	13	59	20.95	1434	13
GART	23	1.140	0.298	2=	370	1.892	9.934	6	365	2.949	39.31	7	122	5.535	157.4	7
MGAR	374	0.139	18.43	1	370	0.417	6.862	1	367	0.556	3.698	4	355	1.168	0.424	2=

Table 15: *Government Bond(Risk Management Evaluation 5% VaR)*

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	3	15.88	943.6	11	1	15.22	883.0	11	1	14.30	801.6	10	1	12.99	689.4	10
LTM	23	3.922	77.49	5=	22	3.477	58.93	6	396	3.561	62.28	4	428	3.255	50.33	4
MA5	3	6.620	220.3	9	1	6.982	242.6	9	1	6.064	187.3	7	3	6.008	184.1	7
MA100	22	4.729	115.2	1	21	4.506	104.3	2	21	4.006	81.16	2	14	3.950	78.70	2
AR1	23	3.922	77.49	5=	22	3.449	57.83	5	396	3.477	58.93	5	428	3.227	49.29	5
ARAIC	23	3.922	77.49	5=	22	3.755	70.33	4	396	3.421	56.74	6	428	3.282	51.38	3
GAR	23	4.423	100.3	2	21	4.840	120.8	1	21	4.590	108.4	1	4	5.341	146.9	6
GJR	23	3.950	78.70	8	21	6.314	201.9	8	21	7.566	280.0	9	7	9.013	378.9	9
EGAR	22	6.843	233.9	10	2	13.29	715.2	10	2	16.55	1005	12	2	18.03	1144	12
EMA5	3	16.97	1044	12	1	16.58	1008	12	1	16.05	958.9	11	3	14.55	823.6	11
EMA100	2	23.70	1722	13	1	22.73	1619	13	1	21.92	1534	13	1	19.58	1297	13
GART	22	4.117	86.13	3=	20	4.451	101.7	3	21	3.978	79.93	3	4	4.423	100.3	1
MGAR	22	4.117	86.13	3=	20	5.257	142.4	7	21	6.147	192.1	8	12	6.926	239.2	8

Table 16: *Equities(Risk Management Evaluation 5% VaR)*

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	9	15.55	913.2	11	30	15.08	870.6	11	30	13.77	755.7	10	64	13.27	712.9	9=
LTM	23	3.755	70.33	5	30	4.701	113.8	1	400	3.644	65.69	1	122	3.866	75.07	3
MA5	16	6.314	201.9	9	74	7.733	290.9	9	74	7.010	244.4	6	113	7.371	267.4	5
MA100	23	4.339	96.38	2	30	5.369	148.4	4	126	5.090	133.6	4	118	5.508	155.9	4
AR1	23	3.811	72.69	4	30	4.618	109.7	2	400	3.616	64.54	2	122	3.922	77.49	2
ARAIC	23	4.395	99.00	1	30	4.506	104.3	3	127	3.449	57.83	3	118	3.950	78.70	1
GAR	23	3.588	63.41	6=	30	6.064	187.3	5	121	6.815	232.3	5	114	8.818	365.1	6
GJR	23	3.588	63.41	6=	30	7.650	285.5	8	119	9.374	404.9	9	108	13.27	712.9	9=
EGAR	15	6.426	208.6	10	30	13.38	722.3	10	30	17.64	1107	12	30	18.94	1234	12
EMA5	16	16.41	992.2	12	30	15.16	878.1	12	44	16.13	966.5	11	108	14.79	845.8	11
EMA100	9	22.70	1616	13	30	22.17	1560	13	68	21.75	1517	13	65	20.39	1378	13
GART	23	3.978	79.93	3	30	6.092	188.9	6	121	7.177	254.9	7	114	9.013	378.9	7
MGAR	16	3.477	58.93	8	30	6.592	218.6	7	44	7.622	283.7	8	108	10.43	483.8	8

Table 17: ***Commodities(Risk Management Evaluation 5% VaR)***

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	1	19.11	1250	12	1	21.53	1494	12	55	22.17	1560	10	45	24.81	1842	8
LTM	1	4.061	83.63	8	36	4.506	104.3	3	36	4.951	126.4	1	36	6.314	201.9	1=
MA5	1	8.484	341.8	10	36	8.957	374.9	6	36	10.54	492.3	5	50	13.57	738.9	5
MA100	1	4.701	113.8	3	36	4.896	123.6	1	36	5.647	163.6	4	36	7.483	274.6	4
AR1	1	4.618	109.7	5	36	4.562	107.0	2	36	4.784	117.9	3	36	6.370	205.2	3
ARAIC	1	4.534	105.7	6	36	4.478	102.9	4	36	4.812	119.4	2	36	6.314	201.9	1=
GAR	1	4.951	126.4	1	36	12.63	659.1	7	36	20.95	1434	8	36	30.35	2469	10
GJR	36	4.312	95.08	7	36	15.83	938.5	10	36	24.45	1803	12	36	32.16	2684	12
EGAR	1	6.787	230.5	9	36	14.74	840.8	9	36	21.28	1468	9	36	27.82	2177	9
EMA5	1	14.72	838.4	11	36	17.33	1078	11	36	18.97	1236	7	36	22.03	1546	7
EMA100	1	24.84	1845	13	36	29.40	2359	13	36	31.13	2561	13	36	32.38	2711	13
GART	1	4.673	112.4	4	36	13.57	738.9	8	36	22.56	1601	11	36	31.15	2565	11
MGAR	1	4.784	117.9	2	36	7.177	254.9	5	36	10.65	500.8	6	36	15.52	910.7	6

Table 18: ***Portfolio(Risk Management Evaluation 5% VaR)***

Steps		***1***				***5***				***10***				***20***		
Models	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*	*TUFF*	*FT*	*UCF*	*Rank*
RW	12	15.66	923.3	11	12	15.27	888.0	11	36	14.55	823.6	10	36	13.38	722.3	9
LTM	23	3.922	77.49	6	342	3.505	60.04	2	352	3.060	43.21	3	342	2.865	36.48	4
MA5	16	6.871	235.7	9	74	6.843	233.9	8	68	5.925	179.3	6	116	7.121	251.4	6
MA100	23	4.534	105.7	3	30	4.451	101.7	1	352	3.644	65.69	1	342	4.117	86.13	1
AR1	23	4.590	108.4	2	342	3.394	55.65	4	352	3.115	45.20	2	342	3.004	41.25	3
ARAIC	23	4.868	122.2	1	342	3.477	58.93	3	352	3.004	41.25	4	342	3.060	43.21	2
GAR	23	4.284	93.78	5	74	5.675	165.1	7	69	7.872	300.2	8	116	11.96	604.3	8
GJR	23	3.755	70.33	7	30	12.79	673.0	9	44	17.11	1057	11	60	21.25	1465	12
EGAR	16	7.844	298.4	10	30	12.99	689.4	10	68	13.55	736.6	9	68	15.74	930.9	10
EMA5	3	23.62	1713	12	29	21.70	1511	12	36	21.22	1463	12	113	20.75	1414	11
EMA100	1	29.32	2349	13	29	26.40	2017	13	36	26.59	2038	13	59	24.73	1833	13
GART	23	4.395	99.00	4	74	5.563	158.9	6	69	6.759	228.8	7	116	10.13	460.5	7
MGAR	23	0.640	2.346	8	370	0.695	1.638	5	366	1.029	0.013	5	119	2.754	32.83	5