Quantifying observation error correlations in remotely sensed data

Conference or Workshop Item

Published Version

Presentation slides

Quantifying observation error correlations in remotely sensed data

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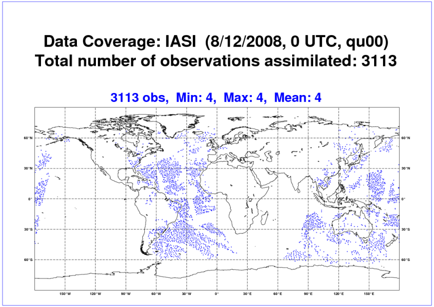
Talk structure

- ★ Introduction and theory
  - Motivation
  - Variational data assimilation
  - Observation error covariance matrices
- ★ Quantifying observation error correlations
  - Desroziers' method of statistical approximation
  - Application to IASI data
  - Results
- ★ Modelling observation error correlation structure
  - Approximate structures for R

What are observation error correlations?

Every observation *y* of a atmospheric variable *x* has an associated error ϵ: $y = Hx + \epsilon$
$\rightarrow$ observation error correlations are present when components of the error vector ϵ are related
$\rightarrow$ measurement errors are attributed to 3 sources: instrument noise, forward model error and representativity error



Error sources

- Instrument noise
  - temperature converted neδt value
  - regular calibrations ensure noise is uncorrelated between channels
- Forward model error
  - errors in discretisation of radiative transfer equation
  - errors in mis-representation of gaseous contributors
  - errors from undetected cloud
- Representativity error
  - contrasting model and observation resolutions
  - observations resolve spatial scales or features that the model cannot
  - contributes to cross channel observation error correlations

Why are correlations important?

Problems

- -ve magnitude and behaviour relatively unknown
- -ve reduce weighting of observations in analysis
- -ve for an observation vector of size 10^6, difficult to store and invert observation error matrix if correlations are included

Benefits

- +ve increase accuracy of gradients of the observed field represented in the analysis
- +ve works with the prior error covariance to specify how observation features should be smoothed
- +ve more information available from observations

Observation error correlation and Shannon Information Content

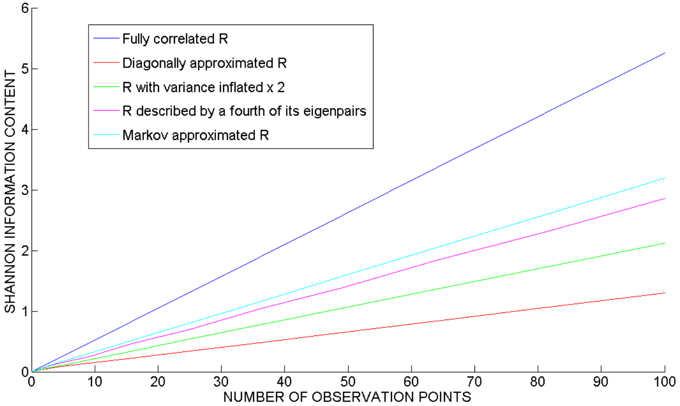


Figure: The *SIC* under different approximations of *R*

Variational data assimilation

Assimilation objective

Model forecast + Observation data → State of atmosphere

Assimilation method Minimise a cost function which measures distance of a solution state x from the observations $y^o \in \mathbb{R}^m$ and the background field $x^b \in \mathbb{R}^n$

Cost Function

$$J(x) \;=\; \frac{1}{2}(x - x^b)^T \mathbf{B}^{-1}(x - x^b) + \frac{1}{2}(y^o - H(x))^T \mathbf{R}^{-1}(y^o - H(x))$$

where **B** and **R** are the background and observation error covariance matrices respectively

An error covariance matrix structure

The observation error covariance matrix takes the form:

$$\mathbf{R} = \mathbf{D}^{1/2}\mathbf{C}\mathbf{D}^{1/2}$$

where **C** is the error correlation matrix

$$\mathbf{C} = \begin{pmatrix} 1 & \rho_{12} & \dots & \rho_{1m} \\ \rho_{12} & 1 & \dots & \rho_{2m} \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{1m} & \rho_{2m} & \dots & 1 \end{pmatrix}$$

and **D** is the error variance matrix

$$\mathbf{D} = \begin{pmatrix} {\sigma^2}_1 & 0 & \dots & 0 \\ 0 & {\sigma^2}_2 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & {\sigma^2}_m \end{pmatrix}$$

A desirable error covariance matrix

Main issue in observation error correlation modelling

- ★ need to calculate matrix-vector product $\mathbf{R}^{-1}(y^o - Hx)$ every time we calculate cost function J
- ★ relatively easy if $\mathbf{R} = \mathbf{D} \equiv m$ scalar multiplications
- ★ **BUT** $y \in \mathbb{R}^{10^6}$ and so $\mathbf{R} \in \mathbb{R}^{10^6 \times 10^6}$ which, if dense, is impossible to store and invert

The perfect partner: what do we want from $\mathbf{R} \neq \mathbf{D}$?

- ♥ structure resulting in an $\mathbf{R}^{-1}$ suitable for storage / can be used cheaply in a matrix-vector product
- ♥ representative of the true error correlation structure
- ♥ greater access to information from the observations and improved analysis accuracy

Quantifying cross-channel correlations: a study

Objective

Generate the true observation error correlation structure for a sample set of remotely sensed data typical of NWP

Data type

- IASI (infrared atmospheric sounding interferometer) observations
- measurements of the infrared radiation emitted by the earth's surface and atmosphere at different wavelengths

Method

We use a post analysis diagnostic derived from variational data assimilation theory [Desroziers, 2005]

Desroziers' method of statistical approximation

Recall the background state, x_b, and observation vector, y, are approximations to the true state of the atmosphere, x_t,

$$\begin{aligned} y &= Hx_t + \epsilon^o \\ x_t &= x_b + \epsilon^b \end{aligned}$$

where ϵ^o and ϵ^b are the observation and background errors respectively.

The best linear unbiased estimate of the true state, x_a, is given by

$$\begin{aligned} x_a &= x_b + \mathbf{K}(y - Hx_b) = x_b + \mathbf{K}d_b^o \\ \mathbf{K} &= \mathbf{BH}^T(\mathbf{HBH}^T + \mathbf{R})^{-1} \end{aligned}$$

Desroziers' method of statistical approximation

Innovation vector

$$
\begin{aligned}
d_b^o = y - Hx_b &= Hx_t + \epsilon^o - Hx_b \\
&\approx \epsilon^o + \mathbf{H}\epsilon^b
\end{aligned}
$$

Analysis innovation vector

$$
\begin{aligned}
d_a^o = y - Hx_a &= y - H(x_b + Kd_b^o) \\
&\approx (\mathbf{I} - \mathbf{HK})d_b^o \\
&\approx \mathbf{R}(\mathbf{HBH}^T + \mathbf{R})^{-1} d_b^o
\end{aligned}
$$

Desroziers' method of statistical approximation

Taking the expectation of the cross product of d_a^o and d_b^o, and assuming

$$\mathbb{E}[\epsilon^o(\epsilon^b)^T] = \mathbb{E}[\epsilon^b(\epsilon^o)^T] = 0,$$

we find a statistical approximation for the observation error covariances

$$\begin{aligned}
\mathbb{E}\left[d_a^o(d_b^o)^T\right] &\approx \mathbb{E}\left[\mathbf{R}(\mathbf{HBH}^T + \mathbf{R})^{-1} d_b^o(d_b^o)^T\right] \\
&\approx \mathbf{R}(\mathbf{HBH}^T + \mathbf{R})^{-1}\mathbb{E}\left[(\epsilon^o + H\epsilon^b)(\epsilon^o + H\epsilon^b)^T\right] \\
&\approx \mathbf{R}(\mathbf{HBH}^T + \mathbf{R})^{-1}(\mathbf{R} + \mathbf{HBH}^T) \\
&\approx \mathbf{R}
\end{aligned}$$

Application to IASI data

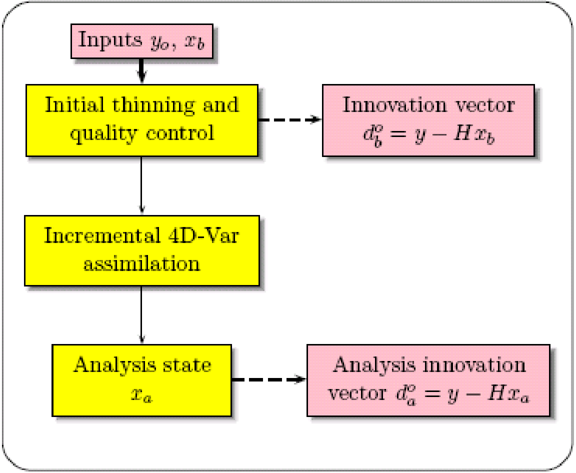
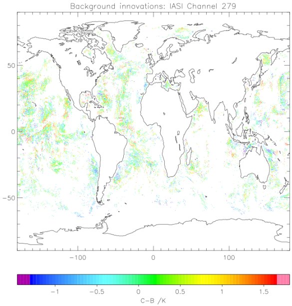


Figure: Assimilation process

Application to IASI data

Methodology

- aim to identify correlations between 139 IASI channels used in 4D-Var assimilation
- only use clear sky, sea surface observations from night and day
- **R** matrix is calculated using $\mathbb{E}\left[d_a^o(d_b^o)^T\right]$



Observation error correlation matrix

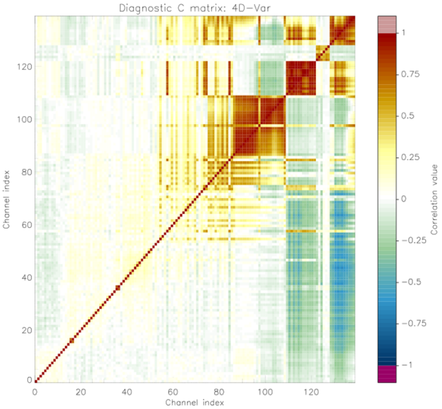
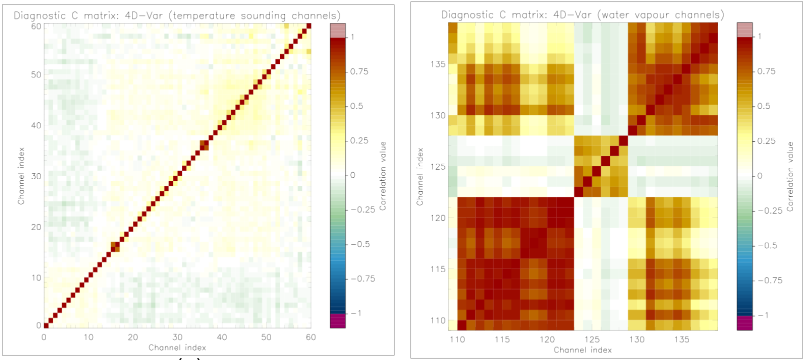


Figure: Error correlation matrix for 139 channels used in Var

Observation error correlation matrix



(a) (b)

Figure: Error correlation matrix for (a) temperature sounding channels; (b) water vapour channels

Operational and diagnosed error variances

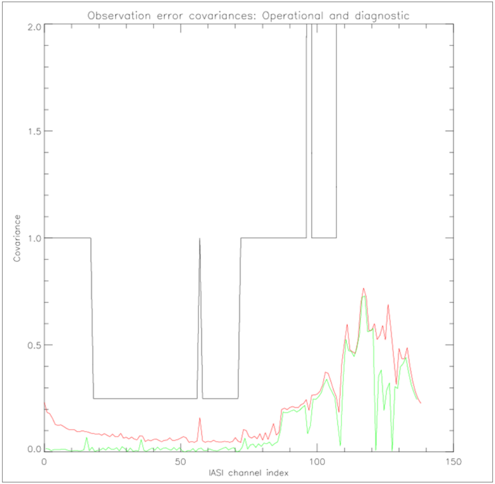


Figure: Operational error variances (black line), diagnosed error variances (red line), and first off-diagonal error covariance (green line)

Diagnosed error variances: comparison with Hollingsworth-Lonnberg (H-L) method

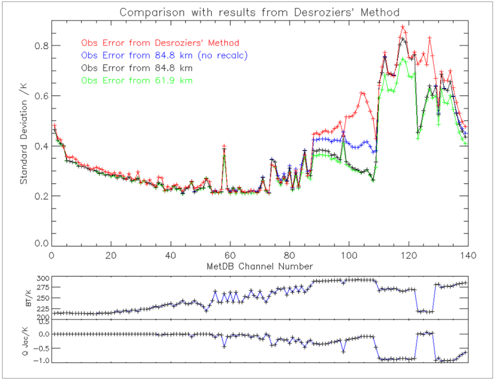


Figure: Diagnosed error variances (red line), H-L diagnosed error variances for 84.8km (blue and black line) and 61.9km (green line) separation. Plot provided by James Cameron, UK Met Office.

Quantifying cross-channel correlations: a summary

- ★ Strong off-diagonal correlations are present between channels with similar spectral properties
- ★ Channels highly sensitive to water vapour have large observation error variances and covariances
- ★ The observation error variance is being overestimated in current asimilation algorithms
- ★ Diagnosed error variances are comparable with those using the H-L diagnostic
- ★ Non-symmetric matrices! → future work

Modelling error correlation structure

What next?

Investigate how to approximate the true error correlation structure within operational assimilation methods...

Current approaches

- a diagonal matrix approximation
- diagonal variance inflation

Alternative approaches

- a Markov error covariance approximation
- a truncated eigendecomposition approximation [Fisher, 2005]
- a Toeplitz to circulant matrix approximation [Healy, 2005]

A Markov error covariance approximation

Consider a Markov covariance matrix of the form

$$R_{ij} = \sigma^2 \rho^{|i-j|}, \quad \rho = \exp\left(-\frac{\delta z}{h}\right)$$

where σ^2 is the error variance, δz is the level spacing, and h is the length scale

This is equivalent to a correlation matrix of the form

$$C = \begin{pmatrix} 1 & \rho & \rho^2 & \dots & \rho^n \\ \rho & 1 & \rho & \dots & \rho^{n-1} \\ \rho^2 & \rho & 1 & \dots & \rho^{n-2} \\ \ddots & \ddots & \ddots & & \ddots \\ \rho^n & \dots & \rho^2 & \rho & 1 \end{pmatrix}$$

A Markov error covariance approximation

The benefit of this choice is that C has a tri-diagonal inverse

$$C^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho & 0 & \dots & 0 \\ -\rho & 1+\rho^2 & -\rho & \dots & 0 \\ 0 & -\rho & 1+\rho^2 & \dots & 0 \\ \ddots & \ddots & \ddots & & \vdots \\ 0 & \dots & -\rho & 1+\rho^2 & -\rho \\ 0 & \dots & 0 & -\rho & 1 \end{pmatrix}$$

and therefore as does R: $R^{-1} = \frac{1}{\sigma^2} I \times C^{-1}$

No need to store and invert R!

An eigendecomposition approximation

Describe C by a truncated eigendecomposition using its leading eigenpairs

$$\tilde{R} = D^{1/2}\tilde{C}D^{1/2} = D^{1/2}\left(\alpha I + \sum_{k=1}^{K}(\lambda_k - \alpha)v_k v_k^T\right)D^{1/2}$$

where (λ_k, v_k) is an eigenvalue, eigenvector pair of C, K is the number of eigenpairs used, and α is chosen such that trace($\tilde{R}$)=trace(D) [Fisher, 2005]

This matrix also has an easily attainable inverse

$$\tilde{R}^{-1} = D^{-1/2}\left(\alpha^{-1} I + \sum_{k=1}^{K}(\lambda_k^{-1} - \alpha^{-1})v_k v_k^T\right)D^{-1/2}$$

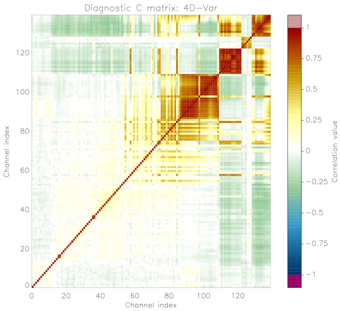
No need to store and invert R!

Summary

- ★ Observation error correlations are often created because of contrasting model and observation resolutions
- ★ Including observation error correlation structure can increase analysis accuracy and information content
- ★ In IASI data, observation error correlations are strongest bewteen channels with similar spectral properties
- ★ In IASI data, the largest observation error covariances are between channels highly sensitive to water vapour
- ★ In order to include observation error correlation structure in data assimilation algorithms, the **R** matrix must be suitably structured

Future work

- ★ Working with a symmetric matrix, eg. fitting a correlation function to the data, taking the symmetric part
- ★ Investigation using the diagnostic update in a identical twin 1D shallow water model experiment



References

- G. Desroziers and L. Berre and B. Chapnik and P. Poli. *Diagnosis of observation, background and analysis-error statistics in observation space*. Q.J.R.Meteorol.Soc., 131, 2005.
- M. Fisher. *Accounting for Correlated Observation Error in the ECMWF Analysis*. ECMWF Technical Memoranda, MF/05106, 2005.
- S.B. Healy and A.A. White. *Use of discrete Fourier transforms in the 1D-Var retrieval problem*. Q.J.R.Meteorol.Soc., 131, 2005.